The effect of unsteady crosswind forces on train dynamic behaviour

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ABSTRACT

This paper sets out a method for the calculation of unsteady cross wind forces on trains, through a process involving the simulation of wind time series and the transformation of these wind time series into aerodynamic force and moment time series through the use of aerodynamic weighting functions. Specific emphasis is given to the method of applying stationary wind tunnel data to the moving full scale situation. An application of this method to a practical situation is then described - the calculation of the relative movement between train pantographs and the overhead electricity supply in high cross winds.

1. INTRODUCTION

 Whilst not common, a number of incidents of train overturning in high winds have been reported from around the world. A great deal of work has been carried out in recent years to determine the risk of such accidents occurring and to develop suitable alleviation methods – in the UK for the Advanced Passenger train in the late 1970’s, and more recently to investigate the crosswind stability of the Class 390 Pendolino train (Baker et al, 2004). A number of pan-European projects have also studied the
problem - TRANSAERO, RAPIDE, DEUFRAKO and AOA, and considerable progress has recently been made in the development of European standards and TSI documentation in this field (CEN (2008), TSI (2006)]. Work is also underway out in Japan and Korea (Fujii et al, 1999). Train overturning is not however the only problem caused by high crosswinds. For example the train body can be deflected sideways on its suspension system that can, in extreme circumstances, result in the infringement of the kinematic envelope, or can cause the pantograph (the electrical current collection system) to deflect such that it loses contact with the overhead transmission wire - the so-called pantograph dewirement problem.

To make proper assessments of these effects, it has come to be realised that the static overturning calculations used in earlier investigations are not wholly adequate, and that some account has to be taken of the train dynamic systems, and some simulation of the unsteady nature of wind gusts is also required. Currently a number of alternative recommendations are made concerning how such calculations are carried out – from the use of relatively simple linearised models, through the use of a typical, average, gust profile (the so called “Chinese” hat profile”) (TSI, 2006) in multi-body simulations, to the use of simulated wind fields on full dynamic models of vehicles (Cheli et al, 2003).

It is the purpose of this paper to address some of the fundamental issues associated with work of this sort. It describes in some detail how the fluctuating crosswind force and moment time histories on trains can be specified and calculated. The outline of the calculation procedure is as follows.

- The specification of the wind field seen by a train moving along a track, through the development if a relatively simple computational method.
- The specification of the aerodynamic admittances and weighting function on which the calculation method is based, through an analysis of wind tunnel experimental results.
- The transformation of the admittances and weighting functions that were obtained from static wind tunnel experiments into a form suitable for the application to moving vehicles in the full scale situation.
- The calculation of the aerodynamic forces through the convolution of the velocity time histories with the weighting functions.

Each of these steps is briefly considered in what follows. In addition this paper will address the specific application mentioned above - the calculation of the displacement between the train pantograph and the overhead wire system in cross winds, which has implications for train design.

2. THE SPECIFICATION OF THE WIND FIELD SEEN BY A MOVING TRAIN

We make the basic assumption that the train is travelling along a section of track, with the mean wind direction normal to the track. The mean wind velocity relative to the vehicle $V$ and the angle of the wind relative to the train ($\Psi$) are then given by

\[ \bar{V} = (v^2 + \bar{u}^2)^{0.5} \]
\[ \tan \Psi = \frac{\bar{u}}{v} \]  

where $\bar{u}$ is the mean wind speed and $v$ is the train speed. To examine the effect of unsteady cross winds on train forces and moments we need to generate an unsteady cross wind field as experienced by the train. Ding et al (2008) carried out such a procedure by numerically simulating wind time histories at a large number of points, separated by short distances, along a track, using complex methodologies to ensure that the time series at each point have the correct spectral characteristics, and correlations between the time series at adjacent points have statistics that are consistent with those measured at full scale. This approach proved to be robust and easy to use, although very computationally intensive, with the result that either only short sections of track could be simulated, or that the simulation points had to be widely separated. In this paper however we take a different approach, and simulate one wind time series only, corresponding to the wind velocity at the location...
of the train at any one instance. This is done by a simple decomposition of the wind spectrum relative to the moving train into a series of sinusoidal velocity variations of random phase, and the combination of these time series into the unsteady velocity time series relative to the train. The spectrum that is used is that of Cooper (1985) which is given by

\[
\frac{nS_u}{\sigma_u^2} = \left[ \frac{4(nL' / \bar{V})}{(1 + 70.8(nL' / \bar{V})^5)^{5/6}} \right] \left( \frac{\bar{u}}{\bar{V}} \right)^2 \left[ 1 - \left( \frac{\bar{u}}{\bar{V}} \right)^2 \right] \left( 0.5 + 94.4(nL' / \bar{V})^2 \right)^2
\]

(2)

\( \sigma_u^2 \) is the variance of the unsteady wind velocity and \( L' \) is a modified turbulence length scale. This is essentially a modification of the well known von Karman spectrum in the plane of reference of a moving vehicle. The values of the spectral density at discrete frequencies \( n_j \) are then used to calculate the unsteady wind velocity time series at the position of the train, through the use of the algorithm

\[
u_j = \sum \left[ 2S_u(n_j) \Delta n_j \right]^{0.5} \sin(2\pi n_j t + 2\pi r_j)
\]

where \( r_j \) is a random number between 0 and 1.

3. THE SPECIFICATION OF AERODYNAMIC ADMITTANCES AND WEIGHTING FUNCTIONS

Now from the standard definition for force coefficient, one may assume that the fluctuating force, consisting of a mean value \( \bar{F} \) and a fluctuating value \( F' \), is related to the mean wind velocity \( \bar{V} \) relative to the train and a fluctuating value \( V' \) by the equation

\[
\bar{F} + F' = 0.5 \rho AC_f (\bar{V} + V' )^2 \approx 0.5 \rho AC_f \bar{V}^2 + \rho AC_f \bar{V} V'
\]

(4)

where \( \rho \) is the density of air, \( A \) is a reference area and \( C_f \) is a force coefficient. This is the quasi-steady expression and assumes that force fluctuations follow velocity fluctuations. Although it is a good approximation in many circumstances, in reality the quasi-steady assumption does not hold completely, and the force fluctuations do not completely follow the velocity fluctuations as the small scale turbulence in the oncoming wind is not fully correlated over the entire exposed area of a vehicle of a train. To allow for this we introduce the concept of the aerodynamic weighting function \( h_f \) and the second term in equation (4) becomes

\[
F' = \rho AC_f \bar{V} \int_0^\infty h_f(\tau)V'(t - \tau) d\tau
\]

(5)

The weighting function thus allows the time history of a fluctuating force to be obtained from the time history of a fluctuating velocity i.e. it is a time domain operator. It effectively weighs the contribution of fluctuating velocities over the preceding time period to the fluctuating forces. Essentially it is calculated from its frequency domain equivalent, the aerodynamic admittance, \( X_f \). This parameter is effectively a normalised ratio of the force spectrum to the wind spectrum and is more easily measured in experiments. It is found that to a good approximation the admittance can be described by

\[
X_f^2 = \frac{1}{(1 + (\pi / \pi')^2)^2}
\]

(6)

where \( \pi = nL / \bar{V} \) is a normalised frequency, \( L \) is the vehicle length and \( \pi' \) is a parameter found from experiments. From wind tunnel results (Baker, 2009) the parameter can be given by a relatively simple curve fit of the form
\[ \tau' = \gamma \sin \psi \]  

where \( \gamma = 2.0 \) for side force coefficients for a variety of trains and 2.5 for lift force coefficients. Through a Fourier Transform of equation (6) we arrive at the following simple form for the weighting function

\[ h_p = (2\pi \tau')^2 e^{-2\pi^2 \gamma} \]  

where \( \tau = \tau' \sqrt{L} \) is a normalised time.

4. TRANSFORMATION TO THE MOVING VEHICLE CASE

The normalisation used in the formulation of the above expressions involved the wind velocity relative to the train \( \bar{V} \). However the various experimental parameters are obtained from wind tunnel tests where effectively \( \bar{V} \) is assumed to be equal to the mean wind tunnel velocity \( \bar{u} \). In applying these results to the moving vehicle case it is necessary first to assume that the relevant normalisation velocity is indeed the wind velocity relative to the vehicle \( \bar{V} \). There is another difference however between the stationary and the moving cases that is illustrated by the velocity vectors of figure 1. This shows that for the stationary vehicle case, longitudinal turbulence fluctuations result in only force fluctuations, whereas for the moving vehicle case, such fluctuations result in both force and yaw angle fluctuations. It is possible to show that in such circumstances that equation (5) above needs to be replaced by the expression

\[ F' = \rho AC_p \left[ 1 + \frac{1}{2C_F} \frac{dC_F}{d\Psi} \right] \Psi \int_0^\infty h_p(\tau)u'(t - \tau) d\tau \]  

5. CALCULATION OF AERODYNAMIC FORCES

From the above equations the following expression can be written for the unsteady force on moving vehicles
\[ F' = \rho A C_F \pi \left( 1 + \frac{1}{2C_F} \frac{dC_F}{d\psi} \cot \Psi \right) \int_0^{\infty} \left( 2\pi \frac{V}{L} \right)^2 e^{-2\pi \frac{V}{L}r} u'(t - \tau) d\tau \]  

(11)

The only parameters that are still to be specified are the force coefficients. Data is available for the side and lift force coefficients for a variety of different types of train. For the purposes of this paper the side and lift force coefficients are approximated by

\[ C_F = K_F \sin \psi \]  

(12)

Whilst these expressions are only rather crude approximations for most vehicles (in particular for the lift force at high yaw angles), nonetheless the simplicity of this form will be seen to be convenient in the calculations that follow.

6. SAMPLE CALCULATION

In this section we present a sample calculation of the fluctuating side and lift forces on a vehicle for a mean cross wind of 20m/s and a vehicle speed of 40m/s. We assume a turbulence intensity of 0.15, a turbulence length scale of 50m, a vehicle reference length, height and area of 20m, 3m, and 70m² respectively, and values of the mean force parameters \( K_S \) and \( K_L \) of 0.8 and 0.4 respectively. The simulated time histories of velocity for this situation is shown in figure 2. It can be seen that the wind speed relative to the vehicle exhibits a lower level of fluctuation than the actual wind speed, because of the large, non-fluctuating vehicle velocity component i.e. the turbulence intensity relative to the vehicle is much smaller than that in a stationary frame of reference.

![Figure 2: Simulated wind time series for wind normal to train (mean wind speed = 20m/s, vehicle speed = 40m/s).](image)

The aerodynamic weighting functions are shown in figure 3, and these show a peak at around 0.1s, and fall to zero at around 0.5s. This implies that the unsteady forces on the vehicle are the weighted values of those due to the unsteady wind forces over the previous 0.5s. A 10 second time slice of the simulated side force is shown in figure 4, together with the quasi-steady side force calculation. It can be seen that the simulated time series is effectively a filtered variation of the quasi-steady value – the admittance function, which represents non-coherence of turbulence, effectively filters out all force fluctuations with time periods less than around 0.5s.
Now experiments at both model scale and full scale suggest that the correlation between lift and side forces is far from perfect. For example for the Class 365 e.m.u. results reported in Baker et al (2008), the correlation coefficient between instantaneous side and lift forces was around 0.7 at high yaw angles, and fell to much lower values of around 0.2 to 0.3 at lower yaw angles. The cause of this lack of correlation can be conjectured to be that the fluctuating side and lift forces are dependent in the main on the wind fluctuations on different streamlines around the train (see the discussion of the “significant streamline” in Sterling et al (2008)). Clearly this needs to be allowed for in the simulation in some way, as otherwise unrealistically highly correlated side and lift force time histories will be produced. This is achieved in the calculations that follow by using the same time histories of velocity for the generation of all forces for frequencies of less than a correlation cut off frequency $n_c$, but assuming that the sinusoidal velocity fluctuations generated above this frequency have a different set of random phases in equation (5) for each force component. It is possible to determine representative values of this correlation frequency limit by comparing the correlations of lift and side force with the experimental values reported above. $n_c$ values of around 0.25Hz gave correlation coefficients that were similar to those found in experiments. Figure 5 shows plots of experimental and simulated side forces against lift force using this method for a mean wind speed of 20m/s and a vehicle speed of 40m/s, with a correlation cut off frequency of 0.25 Hz for the simulation. It can be seen that the two figures are similar. Further the spread of data can be clearly seen and the maximum in side force is not always associated with the maximum in lift force and vice versa.
Finally figure 6 shows side forces simulated for a range of different vehicle speeds for a wind speed of 20m/s. The large scale fluctuations can be clearly seen, with the size of these fluctuations falling relative to the mean force as the vehicle speed increases.

Figure 6: Simulated side force time histories for a mean wind speed of 20m/s and different vehicle speeds.

7. APPLICATION TO PANTOGRAPH SWAY CALCULATION

In this section we consider the application of the method outlined above to the case of “pantograph sway” in cross winds. On a train the pantograph is the cantilever arm system that collects the current from the overhead wire for electrical trains – see figure 7 for a definition of the nomenclature that will be used in what follows. This work arose out of an investigation in the UK into the adequacy of two different railway codes of practice. The first specifies the maximum displacement that can be allowed to occur between the pantograph head and the overhead wire, and resulted in a stiff suspension design, whilst the second was concerned with train derailment and required a less stiff suspension. Often it was found that these codes were in conflict, and since there are neither a large number of dewirements due to excessive pantograph movement or a large number of derailments, it was clear
that there was some inconsistency between the designs. In this section we present a hypothetical calculation of pantograph sway for an idealized train to illustrate a method that can be used to address problems such as these.

In essence the method is straightforward and has the following components.

- The calculation of side and lift forces on the train using the method outlined in previous sections.
- The application of these forces within a train dynamic model to calculate the displacement at the top of the train body, and thus at the pantograph base.
- The calculation of the displacement of the pantograph arm due to cross winds and thus, with the pantograph base displacements, a time history of the pantograph top displacements can be calculated.
- The calculation of the overhead wire displacement, and thus the calculation of the relative displacement between the pantograph top and the overhead wire

In the project above to investigate the codification conflict, a commercial train dynamic model was used. Here however we use a simplified model that represents the train dynamic system by mass / spring / damper models of the primary and secondary lateral, vertical and roll suspensions. The parameters for the model are shown in table 1 below.
Table 1 Vehicle parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body mass</td>
<td>21400kg</td>
</tr>
<tr>
<td>Unsprung mass</td>
<td>1375kg</td>
</tr>
<tr>
<td>Bogie sprung mass</td>
<td>2707kg</td>
</tr>
<tr>
<td>Rolling moment arm length</td>
<td>1m</td>
</tr>
<tr>
<td>Primary lateral stiffness / axle</td>
<td>15700000N/m</td>
</tr>
<tr>
<td>Secondary lateral stiffness / bogie</td>
<td>74000N/m</td>
</tr>
<tr>
<td>Secondary lateral damping / bogie</td>
<td>70000Ns/m</td>
</tr>
<tr>
<td>Primary vertical stiffness / axle</td>
<td>359000N/m</td>
</tr>
<tr>
<td>Secondary vertical stiffness / bogie</td>
<td>84000Ns/m</td>
</tr>
<tr>
<td>Secondary vertical damping / bogie</td>
<td>Ns/m</td>
</tr>
<tr>
<td>Pan height</td>
<td>1m</td>
</tr>
</tbody>
</table>

In addition we assume that the track on which the train is running has the following spectra for lateral ($x$) and vertical ($y$) position irregularities, and cross level irregularity ($xy$) between the rails (Li et al, 2005)

\[
S_x = A_x n_x^2 / ((n_x^2 + n_y^2)(n_x^2 + n_z^2))
\]

\[
S_y = A_y n_y^2 / ((n_x^2 + n_y^2)(n_x^2 + n_z^2))
\]

\[
S_{xy} = (A_y / b^2) n_x^2 n_y^2 / ((n_x^2 + n_y^2)(n_x^2 + n_z^2)(n_x^2 + n_y^2))
\]

where $A_x = 2.19 \times 10^{-7}$ m/rad, $A_y = 4.032 \times 10^{-7}$ m/rad, $n_x = 0.206$ rad/m, $n_y = 0.8246$ rad/m, $n_z = 0.438$ rad/m, and $b$, the track half width = 0.75m. From these spectra it is possible to generate time series of rail roughness in a similar way to the calculation of time series of velocity outlined above. (Bouferouk et al, 2008) carried out a detailed calculation of the pantograph deformation in cross winds and showed that for typical pantograph stiffnesses these deformations were very small (of the order of 2 or 3mm and are thus ignored in the following calculations. The displacement at the pantograph top is then given by the sum of the displacements of the pantograph base, and those due to the rotation of the pantograph due to roll at the base. With regard to the overhead wire, this is supported between gantries placed 50m apart, and at the support points the position of the wire is effectively fixed. Cross winds will cause wire deflections between these support points. The question arises as to what is an appropriate wind velocity that should be used in the calculation of the overhead wires – clearly the wires will react to wind gusts of rather longer period and larger scale than those that might effect the forces on a train. In this paper we assume that the relevant velocity is the running 3s average wind velocity experienced by the train, $u_3$ between at any one time, which is an effective filtering of the high frequency gust components, but allows for lower frequency velocity fluctuations. This velocity is then used to calculate the mid span displacement of the wire through the empirical equation based on long term experience in the UK

\[
x_w = 0.0002 u_3^2
\]

The overhead displacement at all other points is then assumed to be a sine curve, with the maximum value given by the mid span value. The results of such a calculation are shown in figures 8 to 11. Figure 8 shows the simulated side and lift forces for the passage of the train along the track for a mean wind speed of 20m/s and a vehicle speed of 75m/s. These show the large wind induced fluctuations and it can be seen that the time series are broadly correlated at low frequencies, but not at
higher frequencies due to the imposed lack of correlation described above. Figure 9 shows the calculated displacement of the pantograph top and Figure 10 shows the displacement of the overhead wire. The regular repeating nature of the trace as the wire deflects between gantries can be clearly seen. Figure 11 shows the relative displacements between the overhead wire and the pantograph top. The maximum relative displacements occur at the gantry positions – ie when the train is displaced by the cross wind but the wire position is restrained. Whilst this is necessarily the case for straight track, it may not necessarily be the case for curved track where the wire position relative to the track position is not constant. In figure 11 the horizontal line shows the limit imposed in the UK on net displacement, and it can be seen that for the parameters considered in this hypothetical calculation, then this limit is exceeded.

Figure 8: Simulated side and lift forces on the train

Figure 9: Calculated pantograph top displacement (sum of pantograph base displacement, roll displacement of pantograph and bending of pantograph)
7. CONCLUDING REMARKS

This paper has presented a method for the specification of the cross wind forces on moving trains. This method utilizes the weighting function approach, with these functions being obtained from curve fits to experimental data for aerodynamic admittances. It is shown that care is required in applying concepts and equations derived for static situations to the moving vehicle case. The method is a wholly time domain method, which makes it suitable to situations with spatially varying geometry—such as the railway situation with variable spacing of pantograph gantries, curves of different types, etc.
REFERENCES