Dynamics of Tall Buildings under Stochastic Wind Load: applicability of Eurocode EN 1991-1-4 procedures 1 and 2

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ABSTRACT

The new generation of tall buildings is higher and more slender than the buildings built before. Additionally, increased use is made of high-strength materials which have a higher strength yet the same stiffness properties as conventional materials. Both aspects increase the sensitivity of tall buildings to dynamic behaviour. The design has to ensure adequate reliability but it also has to ensure that people in the building feel comfortable particularly when subject to wind-induced vibrations. Fulfilling vibration criteria for human comfort is decisive in the structural design of most tall buildings. The new Eurocode EN 1991-1-4 provides two different procedures to calculate the structural factor and the comfort level. This paper gives insight in the background theory of wind-induced vibrations. On the basis of this the two procedures are compared. The conclusion is that procedure 1 might be unsafe and it turns out that procedure 2 is preferable.

1. INTRODUCTION

Modern tall buildings are relatively light and flexible, which makes them sensitive to dynamic loads. In the design of these buildings, two important aspects have to be taken into account: the design...
has to ensure both the safety of the building and the comfort in the building.

Because of the dynamic behaviour of the building, the maximum load effect, such as moment or stress can be increased compared to the situation in which the building would react in a quasi-static way. Therefore, in the building design, a dynamic amplification factor has to be introduced. This means that the wind load is considered from static case, but is increased by the dynamic amplification factor.

Vibrating buildings can cause unsafe feelings of the occupant. Therefore, with respect to comfort, limit values are prescribed for the vibration intensity of tall buildings. In the current design practice, it is important to pay attention to the dynamic behaviour of tall buildings because economic losses as a result of malfunctioning of the buildings can be large. Especially the comfort check is increasingly important. Some reasons for this are:
- The application of high-strength materials. These materials have a higher strength yet the same stiffness properties as conventional materials. This means that buildings are more sensitive for wind-induced vibrations.
- The application of less non-structural components such as separation walls. These components are often applied in such way that they stand free with respect to small building motions. Therefore less damping is present in the building.
- The increased perception of comfort by the users of the buildings. The users do not expect vibrations. If they do, it provides unsafe feelings. From an economical point of view, this has to be avoided.

Human beings are not directly sensitive to displacement or velocity, they are sensitive to forces operating on them. Therefore acceleration has become the accepted criterion for evaluation in standards concerning motion perception. Building codes and guidelines give acceleration design criteria to ensure the serviceability of a structure given its destination and use.

This paper discusses the determination of the dynamic amplification factor and the horizontal accelerations of tall buildings subjected to wind load. The theory used is mainly based on the studies of Davenport (1961, 1967) who was the first to develop a procedure to determine the response to along-wind forces that was based on a stochastic approach. The theory is then compared with the current building code EN 1991-1-4 for wind. This Eurocode contains two procedures dealing with the dynamic response under wind load: procedure 1 is described in Annex B and procedure 2 is described in Annex C. Procedure 1 is merely based on the studies of Solari (1982,1988, 1993a and 1993b) and procedure 2 originates from Dyrbye and Hansen (1999).

2. STRUCTURAL MODEL

Consider a tall building with height $h$ in the $z$-direction and width $b$ in the $y$-direction, loaded by a distributed wind load $q(y,z,t)$ per unit area (see Fig. 1). The building can be modelled using a $n$ degree of freedom system ($ndof$). For this a coupled system of $n$ equations of motion is formulated:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\} = \{F(t)\}$$

(1)

where:

$[M]$ = the mass matrix

$[C]$ = the damping matrix

$[K]$ = the stiffness matrix

$\{\ddot{u}\}$ = the acceleration vector

$\{\dot{u}\}$ = the velocity vector

$\{u\}$ = the displacement vector

$\{F(t)\}$ = the load vector

Applying modal analysis this system can be transformed to $n$ uncoupled equations of motion in generalised coordinates $\eta(t)$, by making use of the eigenvectors (vibration modes) $\Phi_i(y,z)$ with
natural frequencies \( n_i \). The response \( u \) of a structure under dynamic load can now be expressed in a time-dependent linear combination of vibration modes:

\[
u \equiv u(y, z, t) = \sum_i \eta_i(t) \cdot \Phi_i(y, z)
\]  

(2)

with:

\[
u \equiv u(z, y, t) = \text{response depending on place and time}
\]

\[\eta_i(t) = i^{th} \text{generalised coordinate, depending on time}\]

\[\Phi_i(y, z) = i^{th} \text{eigenmode}\]

The \( i^{th} \) generalised coordinate results from the \( i^{th} \) uncoupled equation of motion of the structural system:

\[
m_i \ddot{\eta}_i + c_i \dot{\eta}_i + k_i \eta_i = \int^h_b \Phi_i(y, z) q(y, z, t) \, dy \, dz = F_i(t)
\]  

(3)

where the \( i^{th} \) modal mass is defined by:

\[
m_i = \int^h_b \Phi_i^2(y, z) \mu(y, z) \, dy \, dz
\]  

(4)

with:

\[
\mu = \text{mass per unit area}
\]

\[
c_i = 2\omega_i \zeta_i m_i = i^{th} \text{modal damping constant } (\omega_i = 2\pi n_i)
\]

\[
k_i = \omega_i^2 m_i = i^{th} \text{modal stiffness constant}
\]

\[
\zeta_i = i^{th} \text{modal damping}
\]

\[
n_i = i^{th} \text{natural frequency}
\]

These uncoupled (modal) equations are the basis of the dynamic analysis of buildings under wind load. The equations have the character of single degree of freedom (sdoF) systems. In Fig. 1, a building model has been drawn on the basis of a sdoF analysis for vibration modes \( \Phi_i(y, z) \) with wind pressures \( q_k \) and \( q_l \) in \( (z_k, y_k) \) and \( (z_l, y_l) \) respectively. For buildings, in general the first natural frequency is dominant, however the theory can easily be expanded with the contribution of higher natural frequencies.

![Figure 1: Schematic view of tall building.](image)
3. WIND CHARACTERISATION

Wind is a phenomenon the properties of which such as direction and speed can only be described in statistical terms. Wind load is a fluctuating load which is not constant in time and place. Buildings are modelled to be in a unidirectional wind field. The wind velocity can be characterised by a mean value over a certain period (in the Eurocode this is 10 minutes). Around this mean value the wind is fluctuating continuously. This has been illustrated in Fig. 2. The fluctuating part of the wind load causes the dynamic behaviour of the building. As input for the model the extreme 10-min averaged wind velocity during a reference period is used. This value can be obtained from both measurements in meteorological stations and from wind statistics.

![Figure 2: Wind velocity $v$ as a function of time.](image)

The fluctuating part of the wind load is characterised with a variance spectrum describing how the variance of the stochastic process is distributed over the different frequencies. Here we use the wind spectrum from Eurocode EN 1991-1-4; it is plotted in Fig. 3:

$$S_L = \frac{nS_{\sigma_v}^2}{\sigma_v^2} = \frac{6.8 \cdot f_L}{(1+10.2 \cdot f_L)^{3/3}}$$  

(5)

![Figure 3: Spectrum of the wind velocity fluctuations](image)

On the horizontal axis a dimensionless frequency $f_L = nL/v$ is used, where $n$ is the frequency, $v$ the mean wind velocity and $L$ a length determined by the average size of a wind gust. On the vertical axis,
a reduced spectrum \( S_L = n S_{vv}/\sigma_v^2 \) is used, where \( S_{vv} \) is the variance spectrum of the wind velocity and \( \sigma_v \) is the standard deviation of the wind velocity.

This spectral description of the wind velocity is valid for one point. The coherence, however, is important for determining the wind load on large building façades. The coherence function describes the extent of coherence of the fluctuating wind in time and space. Our interest is in the total response of a building that does not so much depend on the peak wind load at a single point but rather on the level of coincidence of the peak on a certain surface. In the time domain this is expressed by the correlation of the wind gusts and in the frequency domain this is expressed by the coherence. The mutual relationship between fluctuations in two spots on a façade will be larger for large gusts.

Formulas for the coherence as a function of the frequency and the distance between two points have been derived empirically. Davenport (1961) proposed an exponential function that has been adopted by most researchers. For the points \((z_k, y_k)\) and \((z_l, y_l)\) on the building façade this expression for the coherence \( \text{coh}_{z_k, v_k, v_l} \) of the wind velocities becomes:

\[
\text{coh}_{z_k, y_k, v_k, v_l}(n) = e^{-2n \frac{\left( \frac{C_v z_k - z_l^2 + C_z (y_k - y_l)^2}{\Delta} \right)}{vm(z_k) + vm(z_l)}}
\]

where \( C_v \) and \( C_z \) are decay factors, \( vm \) the mean wind velocity and \( n \) the frequency. EN 1991-1-4 employs \( C_z = C_v = 11.5 \). In Fig. 4 the coherence has been plotted as a function of \( n\Delta/\sqrt{vm} \), where \( \Delta \) is the distance between two points, and \( vm(z) \) is the mean wind velocity that depends on the geographical location and the terrain characteristics.

![Figure 4: Coherence of the wind velocity fluctuations](image)

**4. WIND LOADING MODEL**

**4.1 General description of the model**

In this chapter the wind loading model is presented. For the sake of completeness the whole derivitation is shown; this provides a better insight in the differences between procedures 1 and 2 from EN 1991-1-14.

Davenport was the first to introduce this model. The method is depicted in Fig. 5 in the frequency domain. The method starts with the variance spectrum \( S_{vv}(n) \) of the wind velocities, followed by the multiplication with the aerodynamic admittance \( H_a(n) \), a function depending on the frequency \( n \) and the largeness of the loaded façade area \( A \). The load spectrum \( S_{FF}(n) \) follows from \( S_{FF}(n) = |H_a(n)|^2 S_{vv}(n) \). The spectrum of the forces in the structure, \( S_{FvFv}(n) \), then follows where: \( S_{FvFv}(n) = |H_s(n)|^2 S_{FF}(n) \).
In order to translate the model for the wind velocity into wind pressures on the buildings façades, the following assumptions are made:
- The spectrum of the wind-induced pressure in one place depends on the spectrum of the undisturbed wind velocity at the same height, especially for the frequency range where dynamic effects become important. Geurts (1997) has shown that this assumption may result in an overestimation of the loads.
- The lateral and vertical coherence of the wind-induced pressures on the windward (and also on the leeward) side of the building are equal to the lateral and vertical coherence of the upstream longitudinal wind velocity fluctuations. However, it is known from measurements that the coherence of pressures on buildings is higher (Geurts, 1997). Generally, this assumption leads to an underestimation of the overall loads.
- For the coherence of the pressures on the windward and leeward side, the value 1 is applied. This results in an overestimation of the total wind load.

The above assumptions can be summarised by the name ‘lattice’ or ‘wire frame’ model. The resulting load is conservative.

For the local load effect due to the wind, Bernoulli’s law gives:

$$q_w = \frac{1}{2} \rho v^2 = \frac{1}{2} \rho (v_m + \tilde{v})^2 = \frac{1}{2} \rho v_m^2 + \rho v_m \tilde{v} + \tilde{v}^2$$  \(7\)

with:

- $v_m$ = the mean wind velocity
- $\tilde{v}$ = the fluctuating part of the wind velocity
- $\rho$ = the density of the air

The following linearisation is mostly applied:
\[ q_w \approx \frac{1}{2} \rho v_m^2 + \rho v_m \ddot{v} \]  \hspace{1cm} (8)

This linearisation is accurate if \( \ddot{v} / v_m \ll 1 \), that is, if the turbulence is small.

### 4.2 Wind load on a very small surface

Suppose a very small area \( A \), uniformly loaded by one wind gust, for the wind load \( F \) on that area it holds:

\[ F = A \cdot c \cdot q_w \]  \hspace{1cm} (9)

with:

- \( A \) = the area loaded by the wind
- \( c \) = the pressure coefficient
- \( q_w \) = the dynamic wind pressure

The pressure coefficient \( c \) depends on many factors. However in practice, in time constant pressure coefficients are used. For the average value of the force on a small area it holds:

\[ \mu_F = A \cdot c \cdot \mu_{q_w} = \frac{1}{2} \rho \cdot A \cdot c \cdot v_m^2 \]  \hspace{1cm} (10)

and or the standard deviation it holds that:

\[ \sigma_F = \rho \cdot A \cdot c \cdot v_m \cdot \sigma_v \]  \hspace{1cm} (11)

The coefficient of variation is then:

\[ V_F = \sigma_F / \mu_F = 2 \sigma_v / v_m \]  \hspace{1cm} (12)

With the definition of the turbulence intensity \( I_v = \sigma_v / v_m \) this becomes:

\[ V_F = 2I \]  \hspace{1cm} (13)

The expectation of the peak value of the wind pressure in a certain period can be determined by the sum of the mean wind pressure \( \mu \) and some constant \( g \) times the standard deviation \( \sigma \).

An estimation of the wind pressure that is exceeded on the average once in the period \( \Delta t \) is given by:

\[ \hat{q}(w) = \mu(q_w) + g \sigma(q_w) = \frac{1}{2} \rho \cdot \bar{v}^2 + g \cdot \rho \cdot \bar{v} \cdot \sigma(v) = \frac{1}{2} \rho \bar{v}^2 (1 + 2gI_v) \]  \hspace{1cm} (14)

The peak factor \( g \) is defined as (from (Cartwright, 1958)):

\[ g = \sqrt{2 \ln N + 0.6 / \sqrt{2 \ln N}} \]  \hspace{1cm} (15)

where \( N \) is the number of peak values in that period \( \Delta t \). In order to calculate the value of \( N \), the dominant frequency in the wind gusts is needed. This frequency is about 0.24 Hz. For a period \( \Delta t = 10 \) min this provides a peak factor \( g \approx 3.5 \).

### 4.3 Wind load on a large area

The wind load on a large area is subdivided into a mean and fluctuating part.

#### 4.3.1 Mean wind load on a large area

For the mean part of the modal load on an area \( dA \) it holds that:

\[ d\bar{F}_i = \frac{1}{2} \rho \cdot c \cdot v_m^2 \cdot \Phi_i dA \]  \hspace{1cm} (16)
For the mean modal load on the total surface it then follows:
\[
\bar{F} = \int_{A} \left( \frac{1}{2} \rho \cdot c \cdot v_{m}^{2} \cdot \Phi \right) dA = \int_{h}^{b} \left( \frac{1}{2} \rho \cdot c \cdot v_{m}^{2} \cdot \Phi \right) dydz
\]  
(17)

With a standard setting of the wind velocity at the mean wind velocity \(v_{h}\) at reference height \(z_{s}\) and of the pressure coefficient \(c\) at \(c_{z_{s}}\) it is found that:
\[
\bar{F} = \frac{1}{2} \rho \cdot c_{z_{s}} \cdot v_{m,z_{s}}^{2} \cdot \int_{h}^{b} \int_{c_{z_{s}}}^{c_{z_{s}}} \left( \frac{v_{m}}{v_{m,z_{s}}} \right)^{2} \Phi dydz
\]  
(18)

or:
\[
\bar{F} = \frac{1}{2} \rho \cdot c_{z_{s}} \cdot v_{m,z_{s}}^{2} A \cdot J
\]  
(19)

with:
\[
J = \frac{1}{A} \int_{h}^{b} \int_{c_{z_{s}}}^{c_{z_{s}}} \left( \frac{v_{m}}{v_{m,z_{s}}} \right)^{2} \Phi dydz
\]  
(20)

### 4.3.1 Fluctuating wind load on a large area, static consideration

For the fluctuating part of the modal load \(F_{i}\) on a small area \(dA_{k}\) with coordinates \(z_{k}\) and \(y_{k}\) it holds:
\[
d\bar{F}_{i} = c_{k} \cdot \rho \cdot v_{m,k} \cdot \bar{v}_{k} \cdot \Phi \cdot dA_{k} = \left( c_{k} \cdot \rho \cdot v_{m,k} \cdot \Phi \cdot dA_{k} \right) \bar{v}_{k}
\]  
(21)

The covariance spectrum \(S_{q_{k}q_{l}}\) of the wind pressures in the locations \(k\) and \(l\) (see Fig. 1) is:
\[
S_{q_{k}q_{l}} = \rho \cdot c_{k} \cdot c_{l} \cdot v_{m,k} \cdot v_{m,l} \cdot S_{v_{k}v_{l}}
\]  
(22)

with:
\[
c_{k} = \text{pressure coefficient at location (} y_{k}, z_{k} \text{)}
\]
\[
v_{m,k} = \text{mean wind velocity in the point (} y_{k}, z_{k} \text{)}
\]

where \(S_{v_{k}v_{l}}\) is the covariance spectrum of the wind velocities \(\bar{v}_{k}\) and \(\bar{v}_{l}\) defined as:
\[
S_{v_{k}v_{l}} = \text{coh}_{k,l} \sqrt{S_{v_{k}v_{k}} \cdot S_{v_{l}v_{l}}}
\]  
(23)

The reduced wind spectrum \(S_{L}\) (see section 3) was defined as follows:
\[
S_{L} = \frac{nS_{v_{y}}(n)}{\sigma_{v}^{2}}
\]  
(24)

With this, expression (23) becomes:
\[
S_{v_{k}v_{l}} = \text{coh}_{k,l} \sigma_{y} \cdot \sigma_{y} \cdot \frac{S_{L}}{n}
\]  
(25)

The modal wind load is (see expression 3):
\[
F_{i}(t) = \int_{h}^{b} \int \Phi_{i}(y,z) q(y,z,t) dydz
\]  
(26)

with \(q(y,z,t)\) = the fluctuating wind load per unit area
The variance spectrum \( S_{F_i/F_i} \) of the \( i^{th} \) modal load \( F_i(t) \) becomes:

\[
S_{F_i/F_i} = \iiint_{h \times h \times b} \Phi_i(y_k, z_k) \Phi_i(y_l, z_l) S_{q_k q_l} dy_k dy_l dz_k dz_l
\]

(27)

Combining (22), (25) and (27) leads to:

\[
S_{F_i/F_i} = \iiint_{h \times h \times b} \Phi_i(y_k, z_k) \Phi_i(y_l, z_l) \rho^2 c_k c_l \bar{v}_k \bar{v}_l \sigma_{v_k} \sigma_{v_l} \text{coh}_{v_k v_l} \frac{S_L}{n} dy_k dy_l dz_k dz_l
\]

(28)

Rewriting (28) provides:

\[
S_{F_i/F_i}(n) = \left( \rho c_h v_m z_s \sigma_{v_z} A \right)^2 \frac{S_L(n)}{n} X^2_{\Phi_i}(n)
\]

(29)

with \( A = b \times h \) and:

\[
X^2_{\Phi_i}(n) = \frac{1}{A^2} \iiint_{h \times h \times b} c_k v_m c_l v_m \sigma_{v_k} \sigma_{v_l} \Phi_i(y_k, z_k) \Phi_i(y_l, z_l) \text{coh}_{v_k v_l} dy_k dy_l dz_k dz_l
\]

(30)

In the transition (28)-(29) it is assumed that the wind spectrum is independent of the height; this turns out to be a good assumption. The index \( z_s \) indicates a reference height at which the parameters are normalised. Often the parameters \( c_k \) and \( \sigma_{v,k} \) are assumed to be independent of the place. Expression (30) then becomes:

\[
X^2_{\Phi_i}(n) = \frac{1}{A^2} \iiint_{h \times h \times b} v_m v_m \Phi_i(y_k, z_k) \Phi_i(y_l, z_l) \text{coh}_{v_k v_l} dy_k dy_l dz_k dz_l
\]

(31)

The factor \( X^2 \) is a reduction factor and takes into account the fact that the wind gusts do not occur simultaneously on large surfaces.

In most of the literature the vibration mode \( \Phi \) is taken outside the integral; the consequences of this choice will be discussed in chapters 7 and 8. It will appear that this can lead to a serious underestimation of the response. For the solution of integral (31) often empirical formulas are used. The aerodynamic admittance \( a_H \) in \( S_{FF} = H^2 a_{HH} \), see Fig. 4.14, is related to \( X^2 \) via \( H_a = (\rho \cdot c \cdot v_m A) X \). The variance of the fluctuating part of the modal wind load is:

\[
\sigma_{F_i}^2 = \int_{0}^{\infty} S_{F_i/F_i} dn = \left( \rho \cdot c_h \cdot v_m z_s \cdot A \right)^2 \cdot \int_{0}^{\infty} X^2 \frac{S_L(n)}{n} dn \cdot \left( \sigma_{v_z} \right)^2
\]

(32)

Rewriting provides:

\[
\sigma_{F_i}^2 = \left( \rho \cdot c \cdot v_m z_s \cdot A \right)^2 \cdot \Gamma \cdot \left( \sigma_{v_z} \right)^2
\]

(33)

with:

\[
\Gamma = \int_{0}^{\infty} X^2 \frac{S_L(n)}{n} dn
\]

(34)

The standard deviation of the modal load becomes:

\[
\sigma_F = \rho \cdot c \cdot v_m z_s \cdot A \cdot \sqrt{\Gamma} \cdot \sigma_{v_z}
\]

(35)

Comparison of (35) and (11) provides \( \sqrt{\Gamma} \), as the reduction factor due to the small dimensions of the gusts compared with the large surface of the facade.
With help of expressions (19) and (35) the coefficient of variation of \( F \) becomes:

\[
V_F = \frac{\sigma_F}{\mu_F} = \frac{2\sigma_F \sqrt{\Gamma}}{v_m J} = 2I \cdot \sqrt{B^2}
\]

(36)

with:

\[
B^2 = \frac{\Gamma}{J^2}
\]

(37)

This is the theoretical expression for the parameter \( B^2 \) from Eurocode EN 1991-1-4.

With a reduction factor for large surfaces, the peak value of the wind pressure with a certain return period now becomes:

\[
\hat{q}(w) = \mu(q_w) + g\sigma(q_w) = \frac{1}{2} \rho \cdot v_m^2 \cdot g \cdot \rho \cdot v_m \cdot \sqrt{B^2} \cdot \sigma_v = \frac{1}{2} \rho v_m^2 \left( 1 + 2gI \sqrt{B^2} \right)
\]

(38)

4.3.1 Fluctuating wind load on a large area, dynamic consideration

From the theoretical analysis of a sdof system under stochastic load, for the standard deviation of the modal response it results by very good approximation (see Vrouwenvelder (1998) and Steenbergen (2003)):

\[
\sigma_{\eta_i}^2 = \frac{\sigma_{F_i}^2}{k_i^2} + \frac{2\pi^2}{2\delta_i \pi^2} n_i S_{F_i,F_i}(n_i)
\]

(39)

Here, \( \delta_i \) is the logarithmic decrement of the damping. The first part in the equation (39) is the quasi-static response, the second part the dynamic response. Substitution of (29) and (32) into (39) and multiplication with \( k_i^2 \) (inverse of the quasi-static transfer function) provides the variance of the (considered to be static) wind load including the dynamic resonance:

\[
\sigma_{F_i}^2 = \left( \rho \cdot c \cdot A \cdot v_{m,z_i}^2 \right) \left( \sigma_{\eta_i} \right)^2 \left[ \frac{X^2 S_L(n)}{n} d_n + \frac{\pi^2}{2\delta_i} X^2(n_i) S_L(n) \right]
\]

(40)

where \( n_i \) is the natural frequency of the structure that is considered in the modal analysis.

Expression (40) can be rewritten as:

\[
\sigma_{F_i}^2 = \left( \rho \cdot c \cdot A \cdot v_{m,z_i}^2 \right) \left( \sigma_{\eta_i} \right)^2 \left[ \Gamma + \Upsilon \right]
\]

(41)

With the expression for \( \Gamma \) that was already deduced:

\[
\Gamma = \int_0^\infty X^2 S_L(n) \frac{d_n}{n} dn
\]

(42)

and

\[
\Upsilon = \frac{\pi^2}{2\delta_i} X^2(n_i) S_L(n_i)
\]

(43)

Including the dynamic amplification, the coefficient of variation of the wind load is:
\[ V_F = \frac{\sigma_F}{\mu_F} = \frac{2\sigma_r \sqrt{\Gamma + \Upsilon}}{v_m \sqrt{f}} = \frac{2I\sqrt{\Gamma + \Upsilon}}{J} \]  \hspace{1cm} (44)

The term \( B^2 \) was already defined as:
\[ B^2 = \frac{\Gamma}{J^2} \]  \hspace{1cm} (45)

The factor \( R^2 \) is now defined as:
\[ R^2 = \frac{\Upsilon}{J^2} \]  \hspace{1cm} (46)

This is the theoretical expression of the parameter \( R^2 \) from Eurocode EN 1991-1-4.

With dynamic amplification, the peak value of the wind pressure with a certain return period becomes:
\[ \hat{q}(w) = \mu(q_w) + k_p \sigma(q_w) = \frac{1}{2} \rho \cdot \bar{v}^2 + k_p \cdot \rho \cdot v_m \cdot \sqrt{B^2 + R^2} \cdot \sigma(v) = \frac{1}{2} \rho v_m^2 \left(1 + 2k_p \sqrt{B^2 + R^2}\right) \]  \hspace{1cm} (47)

The background response part \( B^2 \) is related to the quasi-static response to the wind load \( F \). The resonant response part \( R^2 \) is related to the dynamic part of the response.

5. CHECK OF STRENGTH AND COMFORT

Combination of (14) and (38) provides the factor \( s \) from Eurocode EN 1991-1-4, it gives the relationship between the values of expectation of the extreme wind pressure with and without the reduction factor:
\[ s = \frac{1}{2} \rho v_m^2 \left(1 + 2gI\sqrt{B^2}\right) = \frac{1 + 2gI\sqrt{B^2}}{1 + 7I} \]  \hspace{1cm} (48)

Combination of (38) and (47) provides the factor \( d \) from Eurocode EN 1991-1-4 it gives the relationship between the values of expectation of the extreme wind pressure with and without the dynamic amplification:
\[ d = \frac{1}{2} \rho v_m^2 \left(1 + 2k_p \sqrt{B^2 + R^2}\right) = \frac{1 + 2k_p \sqrt{B^2 + R^2}}{1 + 7I\sqrt{B}} \]  \hspace{1cm} (49)

Here, \( k_p \) is the peak factor of the dynamic response. EN 1991-1-4 provides a formula for it.

To determine the comfort level, the value of expectation of the accelerations is calculated using expression (39). For the horizontal acceleration it holds:
\[ \ddot{u} = \omega^2 u \text{ with } \omega = k/m \text{ in the sdof analysis} \]  \hspace{1cm} (50)

In terms of the modal response it holds:
\[ \ddot{\eta} = \frac{k_i}{m_i} \eta \]  \hspace{1cm} (51)

For the variances it then holds:
\[ \sigma_{\eta} = \frac{k_i^2}{m_i^2} \sigma_{\eta} \]  \hspace{1cm} (52)
For the determination of the acceleration it turns out that the quasi-static part of the response can be neglected with respect to the dynamic part. Substitution of (29) into (39) provides:

\[
\sigma_{\eta_h}^2 = \left(\rho c_{z_s} v_{m,z_s} \sigma_{v_{z_s}} A\right)^2 \frac{\pi^2 S_L(n_i)X_{\Phi_i}^2(n_i)}{2\delta_i m_i^2}
\]  

(53)

or:

\[
\sigma_{\eta_h} = \frac{\rho c_{z_s} \nu v_{m,z_s}^2}{\mu_i} R_1, \text{ with } R_1 = \sqrt{\frac{\pi^2}{2\delta_i} X_{\Phi_i}^2(n_i) S_L(n_i)}
\]  

(54)

with \( I_v = \sigma_v / \nu_m \) and with a modal mass per unit area \( \mu_i \).

The peak value of the acceleration can be calculated with help of the formulas for \( k_p \) in EN 1991-1-4.

6. EUROCODE EN 1991-1-4 PROCEDURES 1 AND 2

In this chapter the formulas for the structural factor and the acceleration level are briefly presented with some background information.

6.1 Procedure 1 of EN 1991-1-4

The background factor \( B^2 \) allowing for the lack of full correlation of the pressure on the structure surface is calculated as follows:

\[
B^2 = \frac{1}{1 + 0.9 \left(\frac{b + h}{L(z_s)}\right)^{0.63}}
\]  

(55)

The resonance response factor \( R^2 \) allowing for turbulence in resonance with the considered vibration mode of the structure is calculated as follows:

\[
R^2 = \frac{\pi^2}{2 \cdot \delta} \cdot S_L(z_s, n_1, x) \cdot R_h(\eta_h) \cdot R_b(\eta_b)
\]  

(56)

where:

\[
R_h = \frac{1}{\eta_h} - \frac{1}{2 \cdot \eta_h^2} \left(1 - e^{-2 \eta_h}\right) \quad ; \quad R_h = 1 \text{ for } \eta_h = 0
\]  

(57)

\[
R_b = \frac{1}{\eta_b} - \frac{1}{2 \cdot \eta_b^2} \left(1 - e^{-2 \eta_b}\right) \quad ; \quad R_b = 1 \text{ for } \eta_b = 0
\]  

(58)

with:

\[
\eta_h = \frac{4.6 \cdot h}{L(z_s)} \cdot f_L(z_s, n_1, x) \quad \eta_b = \frac{4.6 \cdot b}{L(z_s)} \cdot f_L(z_s, n_1, x)
\]  

(59)

The standard deviation \( \sigma_{a,x} \) of the characteristic along-wind acceleration at the top of the building is:

\[
\sigma_{a,x}(z) = \frac{c_f \cdot \rho \cdot b \cdot I_v(z_s) \cdot v_m^2(z_s)}{m_{1,x}} \cdot R \cdot K_x
\]  

(60)
Here, \( m_{1,x} \) is the mass per unit height and:

\[
K_x = \frac{\int_{0}^{h} v_m^2(z) \Phi_{1,x}(z) \, dz}{v_m^2(z_s) \int_{0}^{h} \Phi_{1,x}^2(z) \, dz}
\]

The peak value of the acceleration can be calculated with help of the formulas for \( k_p \) in EN 1991-1-4.

### 6.2 Procedure 2 of EN 1991-1-4

The background factor \( B^2 \) allowing for the lack of full correlation of the pressure on the structure surface is calculated as follows:

\[
B^2 = \frac{1}{1 + \frac{3}{2} \left( \frac{b}{L(z_s)} \right)^2 + \left( \frac{h}{L(z_s)} \right)^2 + \left( \frac{b}{L(z_s)} \cdot \frac{h}{L(z_s)} \right)^2}
\]

The resonance response factor \( R^2 \) allowing for turbulence in resonance with the considered vibration mode of the structure is calculated as follows:

\[
R^2 = \frac{\pi^2}{2} \cdot \delta \cdot S_L(z_s, n_{1,x}) \cdot K_s(n_{1,x})
\]

where:

\[
K_s(n) = \frac{1}{1 + \sqrt{\left( G_y \cdot \phi_y \right)^2 + \left( G_z \cdot \phi_z \right)^2 + \left( \frac{2}{\pi} \cdot G_y \cdot \phi_y \cdot G_z \cdot \phi_z \right)^2}}
\]

and

\[
\phi_y = \frac{c_y \cdot b \cdot n}{v_m(z_s)}, \quad \phi_z = \frac{c_z \cdot h \cdot n}{v_m(z_s)}
\]

For several mode shapes the parameters \( G \) and \( K \) are given in Table 1.

<table>
<thead>
<tr>
<th>Mode shape</th>
<th>Uniform</th>
<th>Linear</th>
<th>Parabolic</th>
<th>Sinusoidal</th>
</tr>
</thead>
<tbody>
<tr>
<td>G:</td>
<td>1/2</td>
<td>3/8</td>
<td>5/18</td>
<td>4/\pi^2</td>
</tr>
<tr>
<td>K:</td>
<td>1</td>
<td>3/2</td>
<td>5/3</td>
<td>4/\pi^2</td>
</tr>
</tbody>
</table>

The standard deviation \( \sigma_{a,x} \) of the characteristic along-wind acceleration at the top of the building is:

\[
\sigma_a = c_f \cdot \rho \cdot I_v(z_s) \cdot v_m^2(z_s) \cdot R \cdot K_y \cdot K_y / \mu_{ref}
\]

Here, \( \mu_{ref} \) is the mass per unit height. The peak value of the acceleration can be calculated using the formulas for \( k_p \) in EN 1991-1-4.
7. COMPARISON OF EUROCODE FORMULAS WITH THEORY

In this chapter the theoretical formulas for the dynamic response are compared to the expressions from the procedures 1 and 2 from EN 1991-1-4. It will appear that the expressions of procedure 2 are closer to the theoretical formulas and therefore provide more reliable results.

7.1 Background factor B

Both procedures detailed in section 6 provide expressions for B that are independent of the vibration mode φ. From the analysis in section 8 it appears that the influence of φ on B is smaller than on R, so that the error introduced by this simplification is small.

From Dyrbyre and Hansen (1999) it appears that the formula for B in procedure 2 has been derived for a vibration mode that is linear along the height and uniform along the width. Without large errors this can also be applied to other modes.

7.2 Resonance response factor R

7.2.1 Procedure 1 of EN 1991-1-4

Comparison of the theoretical expression for R (46) and expression (56) given by EN 1991-1-4, procedure 1, provides the following relation:

\[
\frac{X^2}{J^2} = R_h R_b
\]

(66)

From the expressions in procedure 1 it results that this is based on the situation in which the vibration mode is uniform along the width of a building. Applying expression (20) for \( J^2 \) with a uniform vibration mode to expression (66) gives:

\[
\frac{X^2}{J^2} = R_h R_b
\]

(67)

Introducing the full expression for \( X^2 \) (31) provides:

\[
\frac{1}{A^2} b^2 \left( \frac{1}{v_m z_s} \int_{h} v_m^2 \Phi_j dz \right)^2 = R_h R_b
\]

(68)

From Solari (1982, 1988, 1993a and 1993b) and the formulas of procedure 1, it results that the following simplification in the formula for \( X^2 \) is used with respect to expression (68):

\[
\frac{1}{A^2} b^2 \left( \frac{1}{v_m z_s} \int_{h} v_m^2 \Phi_j dz \right)^2 = R_h R_b
\]

(69)
Rewriting provides:

\[
\left( \frac{1}{h^2} \iiint \int coh_{iKy} \, dy_k \, dy_l \, dz_k \, dz_l \right) \sqrt{b^2} = R_h \, R_b
\]  

(70)

So:

\[
R_h \, R_b = \frac{1}{h^2} \frac{1}{b^2} \iiint \int coh_{iKy} \, dy_k \, dy_l \, dz_k \, dz_l
\]  

(71)

As can be seen from the formulas above, some simplifications are introduced in the calculation of the quadruple integral for \( X^2 \). The most important simplification is taking the vibration mode \( \Phi \) outside this integral, introducing in this way a separate factor being the integral of \( \Phi \) along the height. This may lead to some errors in the result. In chapter 8 this is studied further.

### 7.2.2 Procedure 2 of EN 1991-1-4

Comparison of the theoretical expression for \( R \) (46) and expression (63) given by EN 1991-1-4, procedure 2, provides the following relation:

\[
\frac{X^2}{J^2} = K_s
\]  

(72)

From Dyrbyre and Hansen (1999) it appears that \( K_s \) provides values that take into account in a theoretical right way the vibration mode \( \Phi \) in the derivation. Therefore the results from procedure 2 will be closer to the theoretical expressions. It also provides the possibility of rightly including the vigouring vibration mode.

### 7.3 Acceleration level

#### 7.3.1 Procedure 1 of EN 1991-1-4

Comparison of theoretical expression (54) for the acceleration level with the formulas from procedure 1 provides the same simplifications as shown in expression (69).

Comparison of the two expressions (54) and (60) provides the following relation:

\[
\frac{R_x}{\mu_i} = \frac{b}{m_{i,xx}} R \cdot K_x
\]  

(73)

Here, \( m_{i,xx} \) is the mass per unit height and \( \mu_i \) is the modal mass per unit area. For a vibration mode that is uniform along the width \( b \) of the building, the latter is defined as (see (4)):

\[
\mu_i = \frac{m_{i,x}}{bh} \int h \Phi_{1,x}^2 \, dz
\]  

(74)

Applying this to (73) and introducing the expression for \( K_x \) provides:

\[
\frac{R_x}{m_{i,xx}} \int h \Phi_{1,x}^2 \, dz = \frac{b}{m_{i,xx}} R \cdot \frac{\int_0^h v_m^2(z) \Phi_{1,x} \, dz}{\int_0^h v_m^2(z) \Phi_{1,x}^2 \, dz}
\]  

(75)
Elaboration provides:

\[ R_l h = R \frac{1}{v_m^2(z_s)} \int_0^h v_m^2(z) \Phi_{1,s} dz \]  

(76)

Squaring the left and the right hand side gives:

\[ \frac{\pi^2}{2\delta_i} X^2(n_i) S_L(n_i) h^2 = \frac{\pi^2}{2\delta_i} S_L(n_i) R_h R_b \left( \frac{1}{v_m^2(z_s)} \int_0^h v_m^2(z) \Phi_{1,s} dz \right)^2 \]  

(77)

\[ X^2(n_i) h^2 = R_h R_b \left( \frac{1}{v_m^2(z_s)} \int_0^h v_m^2(z) \Phi_{1,s} dz \right)^2 \]  

(78)

Applying the simplification done in (69) results in:

\[ R_h R_b = \frac{1}{h^2 b^2} \int_{h,h,b,b} \int_{h,h,b,b} coh_{ij} \ dy_i dy_j dz_k dz_i \]  

(79)

So, also here, the same simplification is done in procedure 1. In chapter (8) the consequences are discussed.

7.3.2 Procedure 2 of EN 1991-1-4

Comparison of the theoretical expression for the acceleration level with the formulas from procedure 2 provides the following relation:

\[ \sqrt{\frac{\pi^2}{2\delta_i} S_L(n_i) X^2(n_i)} = \sqrt{\frac{\pi^2}{2\delta_i} X^2(n_i) K_s \cdot K_j \cdot K_z} \]  

(80)

Rewriting provides:

\[ X = \sqrt{K_s K_j K_z} \]  

(81)

Again, from Dyrbyre and Hansen (1999) it appears that \( K_s \), \( K_j \) and \( K_z \) provide values by taking into account in a theoretical right way the vibration mode \( \Phi \) in the derivation. Therefore the results from procedure 2 will be closer to the theoretical expressions. It also provides the possibility of rightly including the vigouring vibration mode.

8. NUMERICAL COMPARISON OF THEORY AND EUROCODE PROCEDURES

The theoretical formulas given above have been calculated numerically so that comparisons can be made between the theoretical value and the values provides by procedures 1 and 2 of EN 1991-1-4. The comparison is done for several different situations. Building heights of 100 m, 150 m and 200 m are chosen with a width of 40 m. The first natural frequencies are 0.2 Hz, 0.3 Hz and 0.4 Hz, respectively. The first vibration mode can either be linear along the height or parabolic; along the width of the building, the mode is constant. For the calculation of the structural factor \( c_{s,c,d} \) (ultimate limit state), a wind velocity with a large return period is chosen: 25 m/s. For the roughness length, two values are chosen: 0.1 m and 0.01 m. The mass of the building is 7500 kg per unit area (loaded area). The damping is 1% of the critical damping.

In Table 2 the results for \( c_{s,c,d} \) are shown. Table 2 shows that the values for the structural factor \( c_{s,c,d} \) calculated with procedure 2 are much closer to the theoretical values, in all cases the theoretical
values are a little larger but the deviation is quite small. As can be seen from Table 2, procedure 2 gives up to 10% higher values than procedure 1. In the case that the vibration mode is parabolic, the deviation is the largest.

For the calculation of the accelerations (serviceability limit state) in Table 3 a wind velocity with a return period of about one year is chosen: 15 m/s. The other parameters are the same as those that were used for the calculation in Table 2.

Table 2: Comparison of the values for the structural factor.

<table>
<thead>
<tr>
<th>height</th>
<th>n</th>
<th>vb</th>
<th>z0</th>
<th>mode</th>
<th>$c_d$, theory</th>
<th>$c_d$, proc1</th>
<th>$c_d$, proc2</th>
<th>% error proc 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.2</td>
<td>25</td>
<td>0.1</td>
<td>linear</td>
<td>1.09</td>
<td>1.03</td>
<td>1.07</td>
<td>5 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>1.14</td>
<td>1.03</td>
<td>1.13</td>
<td>10 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.01</td>
<td>linear</td>
<td>1.12</td>
<td>1.06</td>
<td>1.10</td>
<td>4 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>1.16</td>
<td>1.06</td>
<td>1.15</td>
<td>9 %</td>
</tr>
<tr>
<td>150</td>
<td>0.3</td>
<td>25</td>
<td>0.1</td>
<td>linear</td>
<td>1.03</td>
<td>0.97</td>
<td>1.00</td>
<td>4 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>1.06</td>
<td>0.97</td>
<td>1.05</td>
<td>8 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.01</td>
<td>linear</td>
<td>1.05</td>
<td>1.00</td>
<td>1.04</td>
<td>4 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>1.09</td>
<td>1.00</td>
<td>1.08</td>
<td>8 %</td>
</tr>
<tr>
<td>100</td>
<td>0.4</td>
<td>25</td>
<td>0.1</td>
<td>linear</td>
<td>1.01</td>
<td>0.95</td>
<td>0.99</td>
<td>3 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>1.04</td>
<td>0.95</td>
<td>1.03</td>
<td>8 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.01</td>
<td>linear</td>
<td>1.04</td>
<td>0.99</td>
<td>1.03</td>
<td>4 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>1.07</td>
<td>0.99</td>
<td>1.06</td>
<td>7 %</td>
</tr>
</tbody>
</table>

Table 3: Comparison of the values for the peak acceleration in [m/s²].

<table>
<thead>
<tr>
<th>height</th>
<th>n</th>
<th>vb</th>
<th>z0</th>
<th>mode</th>
<th>$\hat{a}$, theory</th>
<th>$\hat{a}$, proc1</th>
<th>$\hat{a}$, proc2</th>
<th>% error proc 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>0.2</td>
<td>15</td>
<td>0.1</td>
<td>linear</td>
<td>0.038</td>
<td>0.032</td>
<td>0.039</td>
<td>24 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>0.049</td>
<td>0.036</td>
<td>0.050</td>
<td>39 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.01</td>
<td>linear</td>
<td>0.040</td>
<td>0.034</td>
<td>0.042</td>
<td>23 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>0.052</td>
<td>0.039</td>
<td>0.053</td>
<td>38 %</td>
</tr>
<tr>
<td>150</td>
<td>0.3</td>
<td>15</td>
<td>0.1</td>
<td>linear</td>
<td>0.027</td>
<td>0.022</td>
<td>0.028</td>
<td>24 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>0.035</td>
<td>0.025</td>
<td>0.036</td>
<td>41 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.01</td>
<td>linear</td>
<td>0.029</td>
<td>0.025</td>
<td>0.031</td>
<td>24 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>0.038</td>
<td>0.028</td>
<td>0.039</td>
<td>41 %</td>
</tr>
<tr>
<td>100</td>
<td>0.4</td>
<td>15</td>
<td>0.1</td>
<td>linear</td>
<td>0.022</td>
<td>0.018</td>
<td>0.023</td>
<td>24 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>0.029</td>
<td>0.021</td>
<td>0.029</td>
<td>40 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>0.01</td>
<td>linear</td>
<td>0.025</td>
<td>0.021</td>
<td>0.026</td>
<td>24 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>parabolic</td>
<td>0.032</td>
<td>0.023</td>
<td>0.033</td>
<td>40 %</td>
</tr>
</tbody>
</table>
From Table 3 it results that the values for the acceleration calculated with procedure 2 are very close to the theoretical values, in all cases on the safe side. As can be seen from Table 3 procedure 2 provides much higher values than procedure 1. In the case that the vibration mode is linear, the deviation is about 25%. In the case that the vibration mode is parabolic the deviation is 40%.

The error in procedure 1 when calculating the comfort level is considerable. Human comfort of building occupants is the most important design criterion in relation to wind-induced dynamic behaviour of tall slender buildings, so it is important that the acceleration level is calculated accurately.

In summary, we can say that procedure 2 of EN 1991-1-4 is preferable for the dynamic stochastic analysis of buildings.

9. CONCLUSIONS

For the structural factor and the acceleration level in a building, a comparison has been made between the theoretical formulas and the procedures 1 and 2 from Eurocode EN 1991-1-4. It appears that for the structural factor $c_{sd}$, procedure 2 provides values that are closer to the theoretical values and that are up to 10% higher than those obtained from procedure 1. For the acceleration level, the differences between procedures 1 and 2 are quite considerable. It appears that for all mode shapes, procedure 2 provides values that are much closer to the theoretical values and that are up to 40% higher than the values provided by procedure 1. Procedure 1 may give unsafe values and underestimates the occurring accelerations. The main reason for this is that the mode shape is taken into account in a simplified way: in the expression for the aerodynamic admittance it has been taken outside the integral. In procedure 2, the contribution of the mode shape is taken into account correctly via the factors $G$ and $K$. Because of this, procedure 2 of the Eurocode provides more reliable results. It is concluded that procedure 1 might be unsafe and that procedure 2 is preferable.

REFERENCES
Dyrbye C., Hansen, S.O., Wind Loads on Structures, John Wiley & Sons, 1999
NEN–EN 1991-1-4, Eurocode, Wind Actions on Structures