Limits for the control of wind-loaded slender bridges with movable flaps
Part I: Aerodynamic modelling, state-space model and open-loop characteristics of the aeroelastic system

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ABSTRACT

This paper presents theoretical investigations on stabilisation of slender bridges under wind action. Aerodynamically effective control shields are applied as actuators. The first part of the paper describes the modelling of the uncontrolled aeroelastic system. Two plants are considered: the original aeroelastic system and the actuator-extended, uncontrolled one. Aerodynamic forces are consistently characterised using linear time-invariant transfer elements in terms of rational functions. On this basis, the aeroelastic systems are represented with linear, time-invariant state-space models. Their dynamic characteristics are investigated with the solution of eigenvalue problems. The second part of the paper shows how and within what limits the mentioned system characteristics can be improved through the use of controlled actuators.

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1. INTRODUCTION

In recent years a number of techniques have been investigated to improve the vibration behaviour of bridges especially under wind action, by systematically imposing additional forces. These forces should reduce the system deflection due to wind action and avoid the occurrence of aeroelastic instabilities.

![Figure 1: Two-dimensional aeroelastic system with four aerodynamically effective degrees of freedom.](image)

This two-part paper addresses damping and stabilisation of an aeroelastic system with additional aerodynamic forces. The forces are generated with rotating control shields that are connected to the bridge girder. In contrast to actuators like reaction wheels or gyroscopes, which affect the bridge with inertial or gyroscopic forces (Peil & Kirch 2008a), not only the flutter but also the divergence wind speed of the system can be modified. Moreover, control shields need no minimum mass and hence they only marginally increase the self-weight of the bridge. The forces that are transmitted from the control shields to the bridge girder are generated by the air flow. Provided that a proper design is applied, considerably lower forces are hence necessary for the shield control compared to the other mentioned kinds of actuators.

Aerodynamically effective, movable control surfaces have been used in aerospace engineering to suppress the influences of disturbances on aircraft wings for many years (e.g. Edwards 1977). Their application to bridge decks was investigated by Kobayashi & Nagaoka (1992) and Ostenfeld & Larsen (1992) for the first time. In aerospace engineering, the primary task of a wing is to produce a lifting force. Flaps, as integrated parts of the aerofoil, modify its surface in order to evoke positive effects without increasing the disturbing impact of gusts. With bridges, however, control shields are extra components that augment the area exposed to the wind. They cannot bear any significant payload and can hence not directly fulfil the intrinsic task of a bridge. In addition to motion-induced aerodynamic forces, new gust-induced forces arise simultaneously, which also need to be suppressed. Therefore, aerodynamically effective control shields are generally less effectual for bridges than for aircraft wings. Moreover, control shields need a minimum wind speed to work. They are not suited for damping oscillations in still air. This shortcoming is similar to that of fin stabilisers, which are used to counteract the rolling of ships. Aircraft wings do not possess this disadvantage either.

Here, the theoretically investigated, two-dimensional bridge-like system is equipped with aerodynamically balanced flaps, which are attached to both sides of the bridge girder (Fig. 1) and actively controlled. In contrast to control shields that are located far away from the bridge girder, adjacent flaps modify the flow around the girder effectively and favourably. This two-part paper presents new insights into fundamental stabilisation limits and the ability of gust alleviation, which are derived with mathematically consistent, correct models and tools. Similar systems with actively controlled flaps have been investigated in several publications. A list of journal articles and dissertations is given at the end of this part of the paper. In addition, a number of conference papers by the same authors are available. For the sake of completeness it should be noted that investigations on passively controlled bridges with aerodynamically effective appendages have been published as well (e.g. Preidikman & Mook 1998, Wilde et al. 1999 ; Aslan & Starossek 2008).
2. STATE-SPACE MODEL AND OPEN-LOOP CHARACTERISTICS OF THE PLANT

2.1 Aerodynamic forces

In order to investigate an aerodynamically controlled bridge system with methods of control theory, a realistic and mathematically consistent description of the forces caused by the wind flow around the girder is of particular importance. Usually, wind action is divided into several types of wind loads.

Together with its structural parameters, motion-induced wind forces influence the properties of an aeroelastic system. For a proper representation of the displacement-force relation, a transfer equation with a linear, time-invariant transfer element is used (Peil & Kirch 2008b).

\[ f(p) = G(p) \cdot \xi_s(p) \]  

(1)

In Eq. (1) the aerodynamic force vector is denoted by \( f \), while the vector of the aerodynamically effective degrees of freedom is denoted by \( \xi_s \). The transfer function \( G \) corresponds to the aero-dynamic admittance of motion-induced wind forces. In this pure frequency-domain description, the values are to be regarded as unilateral Laplace transforms. The variable in the frequency domain \( p \) is the reduced complex frequency.

\[ p = \frac{sb}{U} = (\sigma + i\omega)b/U = \beta + ik \]  

(2)

The complex, non-reduced frequency is symbolised by \( s \), \( b \) stands for the half width of the primary bridge cross section according to the system shown in Fig. 1, and \( U \) symbolises the constant horizontal mean wind speed. The following notation according to Küssner & Schwarz (1940) has been used in aerospace engineering since early publications on aerodynamics. A factor \( q_0 \), which includes the air density \( \rho \), is usually separated from the transfer function of motion-induced forces.

\[ f(p) = q_0 \cdot Q(p) \cdot \xi_s(p) \quad , \quad q_0 = \pi \rho b^2 U^2 \]  

(3)

Thus, the matrix \( Q \) is only a function of the reduced complex frequency \( p \). Its elements are called aerodynamic derivatives.

For the two-dimensional example displayed above, the theoretical derivatives based on potential theory are used as proposed by Küssner & Göllnitz (1964) according to Küssner & Schwarz (1940). In the mentioned references the results are derived for a wing-aileron-tab combination, which is typical of aerospace engineering. The geometry and the derivatives of the airfoil can be linearly transformed into the corresponding properties of the system shown in Fig. 1. Transformation details are not listed here. Bridges which aerodynamically effective flaps are applied to are assumed to have streamlined cross sections. Therefore the derivatives of the bridge-flap system are similar to the ones of the flat plate combination based on potential theory. In contrast to the aerodynamic forces derived by Theodorsen & Garrick (1941), which are used in many publications on aerodynamic control of bridges, Küssner & Göllnitz (1964) additionally consider the case of open gaps between the airfoil components. The flow through these gaps is taken into account in the formulas of the corresponding forces. This approach is used here.

The two-dimensional, aeroelastic system with four aerodynamically effective degrees of freedom shown in Fig. 1 can be described with the following detailed equation.

\[ (Lb \quad M \quad M_{c,\text{win}} \quad M_{c,\text{lee}})^T (p) = q_0 \cdot \left( Q_{\alpha jk}(p) \right) \cdot \left( h/b \quad \alpha \quad \alpha_{c,\text{win}} \quad \alpha_{c,\text{lee}} \right)^T (p) \]  

(4)

Forces that act on the total aerodynamically effective cross section are denoted by \( L \) and \( M \). The moments \( M_{c,\text{win}} \) and \( M_{c,\text{lee}} \) act on the flaps around their hinges. A dimensionless \( Q \)-matrix is obtained when using identical dimensions for both the different types of deformations and the different types of loads. For the investigations presented here, the width of the flaps is set to \( b_c = 0.1 \cdot b \). As can be seen in Fig. 1, the flap hinges, which are fixed relative to the bridge girder, are located at the middle of the flaps. The derivatives of the bridge cross section without flaps correspond to the sub-matrix of \( Q \) in Eq. (4) for \( j,k = 1,\ldots,2 \) and \( b_c = 0 \). The conversion to Scanlan derivatives
(Simiu & Scanlan 1996), which are often used in civil engineering, can for purely imaginary frequencies \( p = 0 + ik \) be taken from Peil & Kirch (2008b), for instance.

Gusts act on girder and flaps. Similar to Eq. (1), gust-induced forces \( \mathbf{d}^g \) — also called buffeting forces — which constitute another type of wind forces, can be related to mean-value-free, fluctuating velocity components of gusts \( \mathbf{a}_g \) with aerodynamic gust admittances \( \mathbf{Q}^g \).

\[
\mathbf{d}^g(p) = q_0 \cdot \mathbf{Q}^g(p) \cdot \mathbf{a}_g(p)
\]

For the system given in Fig. 1, according to potential theory (Sears 1940), the vector of the dimensionless gust speeds contains one element only, the dimensionless vertical gust speed \( \mathbf{a}_g = \frac{w_g}{U} \).

Forces due to flow separation and vortex shedding are not considered here. Regarding the streamlined cross section, they are assumed to be negligible.

Usually, analytic functions of the complex frequency are taken to express the transfer function of motion-induced aerodynamic forces. With the aid of these functions, the derivatives of bridge cross sections, which are available for harmonic oscillations, are approximated. Rational functions are the most commonly used analytic transfer function approximations in aerospace engineering as well as in bridge engineering. In this paper, rational functions according to the Minimum-State Method (Karpel 1981) are used.

\[
\mathbf{Q}(p) = \mathbf{A}_0 + \mathbf{A}_1 p + \mathbf{A}_2 p^2 + \mathbf{D}(p \mathbf{I} - \mathbf{R})^{-1} \mathbf{E} p, \quad \mathbf{R} = -\text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_{n_L})
\]

This approach is based on rational transfer functions with single, real poles \((-\gamma_i)\), which are the same for all derivatives. Rational functions are particularly suitable for approximating the aerodynamic derivatives of girders with streamlined cross sections (Peil & Kirch 2008b). The constant matrices \( \mathbf{A}_1, \mathbf{A}_2, \mathbf{D} \) and \( \mathbf{E} \) are determined with elaborate approximation procedures according to Tiffany Hoadley & Adams (1988). For the following examples, the derivative approximation is carried out with \( n_L = 5 \) poles. Steady values of the derivatives are incorporated in the \( \mathbf{A}_0 \) matrix. Exact matching of the steady values is important for the evaluation of the divergence wind speed. In Eq. (6) the parameter \( b_c \) is no longer available. Hence, the derivatives for the flap-free case cannot be obtained as a submatrix of the case with flaps. The approximation is thus separately carried out for the cross section with and without flaps.

As described for motion-induced forces, the approximation of the gust admittance can be performed as well. Separated from the derivatives, it is approximated with its own poles.

2.2 State-space model and characteristics of the aeroelastic plant without flaps

After inserting Eq. (6) into Eq. (3), a part of the last summand of the resulting equation can be transformed into a linear differential equation with constant coefficients by introducing artificial so-called aerodynamic lag states \( \mathbf{\xi}_a \).

\[
\mathbf{\xi}_a = (p \mathbf{I} - \mathbf{R})^{-1} \mathbf{E} p \mathbf{\xi}_a, \quad \mathbf{\xi}_a' = -\mathbf{R} \mathbf{\xi}_a = \mathbf{E} \mathbf{\xi}_a'
\]

The prime ( \( ' \) ) symbolises the generalised differentiation with respect to the non-dimensionalised time \( \mathbf{T} \).

\[
\mathbf{T} = (U/b) t
\]

The time is denoted by \( t \). Together with a linear structure description of the same kind,

\[
\mathbf{M}_s \ddot{\mathbf{x}}_s + \mathbf{C}_s \dot{\mathbf{x}}_s + \mathbf{K}_s \mathbf{x}_s = \mathbf{f} + \mathbf{u} + \mathbf{d}^g + \mathbf{d}^d
\]

the aeroelastic system can be represented by a linear, time-invariant state-space model

\[
\dot{\mathbf{x}} = \mathbf{A} \mathbf{x} + \mathbf{B} \mathbf{u} + \mathbf{E}^g \mathbf{d}^g + \mathbf{E}^d \mathbf{d}^d
\]

\[
\mathbf{y} = \mathbf{C} \mathbf{x}
\]
where \( \mathbf{x} \) is the state vector and \( (\cdot) \) symbolises the generalised differentiation with respect to time \( t \).

\[
\mathbf{x} = \begin{bmatrix} \mathbf{\dot{\xi}}^T \\ \mathbf{\xi}^T \\ \mathbf{\xi}_a^T \end{bmatrix}
\]

(11)

The matrices of the structural variables mass, damping and stiffness are denoted in these equations by \( \mathbf{M}_s \), \( \mathbf{C}_s \) and \( \mathbf{K}_s \) respectively. \( \mathbf{A} \) is the system matrix

\[
\mathbf{A} = \begin{bmatrix} -\mathbf{\bar{M}}^{-1}\mathbf{\bar{C}} & -\mathbf{\bar{M}}^{-1}\mathbf{\bar{K}} & q_0\mathbf{\bar{M}}^{-1}\mathbf{\bar{D}} \\ \mathbf{I} & 0 & 0 \\ \mathbf{E} & 0 & (U/b)\mathbf{R} \end{bmatrix}
\]

(12)

where

\[
\mathbf{\bar{M}} = \mathbf{M}_s - q_0(b/U)^2\mathbf{A}_s, \quad \mathbf{\bar{C}} = \mathbf{C}_s - q_0(b/U)\mathbf{A}_s, \quad \mathbf{\bar{K}} = \mathbf{K}_s - q_0\mathbf{A}_b
\]

(13)

In the state equation (10a) \( \mathbf{d}^g \) and \( \mathbf{d}^d \) represent the gust forces and other disturbance forces respectively. The control input \( \mathbf{u} \) is important for closed-loop control, and is to be regarded in this section as external forces, which correspond to the structural degrees of freedom. The input matrices \( \mathbf{B} \), \( \mathbf{E}^g \) and \( \mathbf{E}^d \) of the control input and the disturbances have the same entries.

\[
\mathbf{B} = \mathbf{E}^g = \mathbf{E}^d = \begin{bmatrix} (\mathbf{\bar{M}}^{-1})^T & \mathbf{0}^T & \mathbf{0}^T \end{bmatrix}^T
\]

(14)

The output equation (10b) cannot extract more than the structural states \( \mathbf{\dot{\xi}}_s, \mathbf{\xi}_s \). In reality, the mathematically introduced lag states \( \mathbf{\xi}_a \) cannot be measured. The mean horizontal wind speed \( U \) occurs in the system matrix \( \mathbf{A} \) as a parameter.

The dynamic characteristics of the aeroelastic system can be evaluated with an eigenvalue analysis of the system matrix \( \mathbf{A} \). System stability is of major interest in this context. Due to the effect of motion-induced aerodynamic forces, aeroelastic instabilities can occur in the form of flutter and divergence. Unless otherwise explained, the two terms should specifically denote the cases of neutral stability. Since the system matrix contains the mean horizontal wind speed \( U \), a parameter-dependent, linear eigenvalue problem must be solved.

### Table 1: Structural properties of the two-dimensional aeroelastic model.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Half deck width:</td>
<td>( b = 15.0 ) m</td>
</tr>
<tr>
<td>Mass:</td>
<td>( m = 25.0 \times 10^3 ) kg/m</td>
</tr>
<tr>
<td>Moment of inertia:</td>
<td>( I = 2.80 \times 10^6 ) kgm(^2)/m</td>
</tr>
<tr>
<td>Eigenfrequencies:</td>
<td>( \omega_h = 0.628 ) 1/s</td>
</tr>
<tr>
<td>( k_h = \omega_h^2 m ); ( k_a = \omega_a^2 I )</td>
<td>( \omega_a = 1.13 ) 1/s</td>
</tr>
<tr>
<td>Logarithmic damping decrements:</td>
<td>( \delta_h = 0.0126 )</td>
</tr>
<tr>
<td>( c_h = \delta_h m \omega_h / \pi ); ( c_a = \delta_a I \omega_a / \pi )</td>
<td>( \delta_a = 0.0126 )</td>
</tr>
</tbody>
</table>

The two-dimensional, generalised system in Fig. 1 is used as an example with the characteristic structural properties given in Table 1. The corresponding structural matrices are as follows:

\[
\mathbf{M}_s = \begin{bmatrix} mb^2 & 0 \\ 0 & I \end{bmatrix}, \quad \mathbf{C}_s = \begin{bmatrix} c_b b^2 & 0 \\ 0 & c_a \end{bmatrix}, \quad \mathbf{K}_s = \begin{bmatrix} k_b b^2 & 0 \\ 0 & k_a \end{bmatrix}
\]

(15)
Neutral stability appears at the zero crossings of the eigenvalue real-part curves (Fig. 2) as flutter at $U = 47.6$ m/s and as divergence at $U = 63.7$ m/s. This identification is possible when inspecting the eigenvalues and state eigenvectors. The indifferent flutter point occurs in two conjugate-complex eigenvectors with complex values. Its eigenfrequencies are purely imaginary and conjugate complex. In the case of the flat plate, both structural degrees of freedom appear in the same order of magnitude, as can be predicted for the classical bending-torsional flutter. Indifferent divergence has only one eigenvector, the element values of which are real and vanish in the velocities $\xi$, and in the lag states $\xi_a$. Its eigenfrequency is zero. In the case of uncontrolled bridges, the divergence wind speed is usually higher than the flutter wind speed and therefore normally not the focus of interest. This statement cannot be held up when actuators are applied. Part II of the paper explains how and within what limits the system characteristics can be changed.

2.3 State-space model and open-loop characteristics of the aeroelastic plant extended with flaps

The mathematical description of the flap-extended plant depends on the type of the chosen control input. One possibility is to act on the extended plant through forces in the form of torques around the flap hinges. In this paper it is assumed that there is no passive coupling between the flaps and the bridge girder. In addition to inertial forces, which arise when accelerating the flaps, the input forces are thus buffered by motion-induced aerodynamic forces only. This usually leads to very non-robust controllers and, together with other reasons, to various numerical problems. The situation becomes worse if the flaps are modelled without a mass. This is done here in order to investigate the purely aerodynamic effectiveness of flaps. The self-weight of the bridge should not unnecessarily increase due to attached flaps. Hence, a lightweight construction is preferable and a massless modelling of the flaps is not unrealistic.

If a control input is chosen in the form of displacements, the mentioned numerical problems are circumvented. For the derivation of the motion equations of an aeroelastic plant with flaps and displacement input, the following procedure is recommendable. In the transfer equations of aerodynamic forces and in the transfer matrices of section 2.1 only those rows are considered that correspond to forces acting on the total aerodynamically effective system. Accordingly, in the approxi-
mation matrices $A_i$ and $D$ the remaining rows must be cancelled. The aerodynamically effective degrees of freedom can be separated. One part still corresponds to the structural degrees of freedom $\xi_s$. The other part represents the displacement inputs $\xi_c$, which, in the example of Fig. 1, include the relative flap angles $\alpha_{c, \text{win}}$ and $\alpha_{c, \text{lee}}$. The derivatives can be separated as well.

$$ f = q_0 \left( Q^s - Q^c \right) \left( \xi_s^T - \xi_c^T \right) = q_0 Q^s \xi_s + q_0 Q^c \xi_c $$  \hspace{1cm} (16)

Motion-induced aerodynamic forces are thus assigned to their different origins. Correspondingly, the approximation matrices must be separated or remain unchanged.

$$ A_i = \left( A_i^s \hspace{0.5cm} A_i^c \right), \hspace{1cm} E = \left( E^s \hspace{0.5cm} E^c \right), \hspace{1cm} D = D^s = D^c, \hspace{1cm} R = R^s = R^c $$  \hspace{1cm} (17)

The transfer from displacement inputs to control forces $u$, which act on the total aerodynamic cross section, can be described with a state-space model

$$ \dot{\xi}_c = A_c \xi_c + B_c x_c $$  \hspace{1cm} (18a)

$$ u = C_c \xi_c + D_c x_c $$  \hspace{1cm} (18b)

where

$$ x_c = \left( \xi_c^T \hspace{0.5cm} \xi_c^T \hspace{0.5cm} \xi_c^T \right)^T. $$  \hspace{1cm} (19)

The matrices of the state-space model can be derived from the approximation matrices of Eq. (17) as follows.

$$ A_c = \frac{U}{b} R^c, \hspace{1cm} B_c = \left( 0 \hspace{0.5cm} E^c \hspace{0.5cm} 0 \right), \hspace{1cm} C_c = q_0 D^c, \hspace{1cm} D_c = q_0 \left( \frac{b}{U} \right)^2 A_2^c - \frac{b}{U} A_1^c \hspace{0.5cm} A_0^c $$  \hspace{1cm} (20)

The motion equations of the flap-extended aeroelastic plant comply in their form with Eq. (9). When applying flaps without a mass the structural matrices $M_s$, $C_s$ and $K_s$ remain unchanged compared to Eq. (15). The forces on the right-hand side of Eq. (9) must be specified as explained in this section. As a consequence of separating the aerodynamically effective degrees of freedom, the approximation matrices $A_i^s$ and $E_i^s$ inside the system matrix $A$ and the aerodynamic lag states $\xi_a$ in the state vector $x$ must be modified with a superscripted index $s$.

Since the derivatives $Q^s$ and $Q^c$ are approximated simultaneously, the aerodynamic lag states can be added and a single new state $\xi_a$ can be defined. If the summand $Bu$ is replaced by $B x_c$ where

$$ \tilde{B} = (B D_c) + \left( 0^T \hspace{0.5cm} B_c^T \right)^T, $$  \hspace{1cm} (21)

both state-space models can be combined into a single one with the following state equation.

$$ \dot{x} = Ax + \tilde{B} x_c + E \xi d^c + E \xi d^d $$  \hspace{1cm} (22)

The state-space model can be extended with summands taking inertia effects of the flaps into account.

When extending the plant with unmoved flaps, its eigenvalue curves are altered (Fig. 3). Since a larger area is exposed to the wind, the flutter wind speed decreases to $U = 43.5 \text{ m/s}$ and the divergence wind speed to $U = 53.1 \text{ m/s}$. To systematically influence the extended aeroelastic system, the displacement inputs of the extended plant must depend on its outputs or states. Hence, the control loop must be closed.

As a consequence of the rational function approximation, the vector $x_c$ contains not only the displacement inputs $\xi_c$, but also their derivatives $\dot{\xi}_c$ and $\ddot{\xi}_c$. These subvectors are not independent of one another and necessitate the application of an actuator model, which is at least mathematically motivated (ZAERO 2004). For this purpose, a state-space model can be used that represents three parallel PT3 elements.
The submatrices only have nonzero entries on the main diagonal.

\[ C_{i,j} = \text{diag}(c_{i,j}) \]  

(25)

The static gain of this actuator is set to one. If the transient response is tuned to be sufficiently fast and non-oscillating, the input \( u_{ac} \) can be transferred to itself and its derivatives with sufficient precision. For a single displacement input, the transfer function is as follows.

\[
G_{ac,k}(s) = \frac{c_{i,k}^0}{s^3 + c_{i,k}^2 s^2 + c_{i,k}^1 s + c_{i,k}^0} = \frac{-s_1 s_2 s_3}{(s-s_1)(s-s_2)(s-s_3)}
\]  

(26)

In this paper its poles are located on the negative branch of the real axis at \( s_1 = -100 \, 1/s \), \( s_2 = -150 \, 1/s \) and \( s_3 = -200 \, 1/s \). Identical poles for different displacement inputs are unproblematic, because they are connected to different eigenvectors.

The actuator model of Eq. (23) can be combined with the state-space model of Eq. (22).

\[
\begin{bmatrix}
\dot{x}
\end{bmatrix}
= \begin{bmatrix}
A & B
\end{bmatrix}
\begin{bmatrix}
x
\end{bmatrix}
+ \begin{bmatrix}
0
\end{bmatrix}
\begin{bmatrix}
u_{ac}
\end{bmatrix}
+ \begin{bmatrix}
E
\end{bmatrix}
\dot{d}
+ \begin{bmatrix}
E
\end{bmatrix}
\begin{bmatrix}
d
\end{bmatrix}
\]

(27a)

\[
y = \begin{bmatrix}
C & 0
\end{bmatrix}
\begin{bmatrix}
x
\end{bmatrix}
\]

(27b)

The dynamic behaviour of sensors and other actuators is neglected in this paper. They are assumed to be fast compared to the components of the extended plant that are investigated here. With linear relations, the displacement inputs can be kinematically coupled to one another. For the numerical example, three different cases are considered: separately driven flaps, flaps rotated in opposite directions \( (\alpha_{c} = -\alpha_{c,lee} = \alpha_{c,win}) \) and flaps rotated in the same direction \( (\alpha_{c} = \alpha_{c,lee} = \alpha_{c,win}) \). The first two cases require information about the flow direction.

The block triangular form of the system matrix in Eq. (27a) shows that the eigenvalue curves described above are not affected by the actuator dynamics. Basis for the following investigations is the state-space model of Eq. (27). To keep the actuator eigenvalues out of the closed loop, a state transformation into the canonical form is performed. In the canonical state space, the actuator eigenvalues can be separated.
CONCLUDING REMARKS

This first part of the paper describes the modelling of a bridge-like system that is extended with flaps. The flaps are attached to both sides of the bridge girder. A general mathematical description is presented and applied to a numerical example that corresponds to a two-dimensional, sectional model of a slender bridge. Motion and gust induced forces are taken into account and described on the basis of potential theory. The transfer functions are approximated with rational functions. Together with a linear description of the structure, rational functions allow the aeroelastic system to be represented with a linear, time-invariant state-space model. In order to avoid numerical problems and non-robust controllers, a control input in the form of displacements is chosen. The characteristics and especially the occurring instabilities of the flap-free and the uncontrolled, flap-extended system are determined. They are important as references for the investigations that are presented in the second part of the paper. The approximation of the motion-induced aerodynamic forces with rational functions necessitates a mathematically motivated actuator model. Based on the derived mathematical representation of the aeroelastic plant, the second part of the paper addresses controller design and the behaviour of the controlled system.

REFERENCES


More references on actively controlled bridges with aerodynamically effective control shields:


