Equivalent Static Wind Loads of an Asymmetric Building with 3D Coupled Modes

Yi Tang and Xin-yang Jin

1 Associate Professor, Wind Engineering Research Center, China Academy of Building Research, Beijing 100013, China, yitang_hunan@163.com
2 Professor, Wind Engineering Research Center, China Academy of Building Research, Beijing 100013, China, jinxinyang@cabrtech.com

ABSTRACT

Recent trends towards developing increasingly taller and complex buildings of irregular geometric shapes imply that these structures are potentially more responsive to wind excitation. Making accurate predictions of wind loads especially equivalent static wind loading is therefore a necessary step in structural design. In this paper, by introducing mode coupling coefficients, a modified SRSS method is proposed to calculate resonant components of the responses and the resonant static-equivalent wind loads. Three dimensional background static-equivalent wind loading is evaluated using Load-Response-Correlation (LRC) method. The background and resonant static-equivalent loads are combined with weight factors. The consequential total equivalent wind load is verified through a 49-storey building with three-dimensional mode shapes.

KEYWORDS: COUPLED MODES; ESWL; MODIFIED SRSS METHOD; LRC METHOD.

Introduction

Modern high-rise structures continue to evolve with unsymmetrical plan forms and irregular vertical arrangements. In addition, these buildings are being built in density developed suburban or urban environments. These trends have led to the effects of unsymmetrical loadings on high-rise structures, commonly referred to torsional effects. The unbalanced load effects can be further exacerbated by non-alignment of center of stiffness of the structural system and center of mass of the floor plates, which result in three-dimensional (3D), coupled mode shapes and experience coupled responses when exposed to wind loads.

Design practice often requires dynamic wind loads on buildings to be modeled as equivalent static wind loads (ESWLs). Accordingly, a relatively simple static structural analysis procedure can be adopted for a more detailed structural analysis and design. The gust response factor (GRF) approach, introduced by Davenport(1967) for along wind excited buildings, is most widely used ESWL evaluation method in building codes and standards. For the past decades, many researchers have developed ESWL calculation method with regard to particular problems. But most methods are concerning one-dimensional wind excited response(Holmes(2002); Solari(1990); Zhou(2000)) (et al alongwind, acrosswind or torsional). Solari(1993) introduced a global model for calculating 3-D ESWLs , but not considering modal coupling and cross correlation between forces along different directions. Chen(2005) proposed a framework to evaluate ESWLs of asymmetric tall buildings undergoing lateral-torsion coupled motion using wind loading measured by high frequency force balance technique.
In this paper, the ESWL of a super-all building of concrete construction with coupled three-dimensional (3D) mode was calculated using pressure data measured in wind tunnel. In evaluating resonant static-equivalent load a mode coupling coefficient is introduced to considerate mode coupling effects and background static-equivalent wind load is calculated by Load-Response-Correlation (LRC) method.

**ESWL analysis based on measured pressure field**

*Quasi-static wind loading*

The LRC method of Kasperski (1992) has been using to determine the equivalent static load distributions of the background component. The LRC method can give an expected equivalent load distribution corresponding to a particular load effect or response. The theory is easily extended to two or three dimensions as presented by Chen(2005). The equivalent wind loading is given by

\[
\hat{F}_{s\ell}(h) = \frac{g_B}{\sigma_{s\ell}(z)} \sum_{l=x,y,\theta} \int_{z_1}^{z_2} Cov_{r_{s\ell}}(h,h) \mu_{s\ell}(z;h_i) dh_i, \quad (\ell = x, y, \theta)
\]

where \(g_B\) is the peak factor for background response; \(\mu_{s\ell}(z;h_i), (s, l = x, y, \theta)\) is the influence coefficient, which represents response in \(s\) direction at \(z\) height with an unit force acting at \(h_i\) height in \(l\) direction; \(Cov_{r_{s\ell}}(h,h)\) is the covariant coefficient between two forces, one at \(h_i\) height in \(l\) direction and the other at \(h\) height in \(\ell\) direction \((\ell, l = x, y, \theta)\); \(\sigma_{s\ell}(z)\) is background component of RMS response in \(s\) direction.

*Equivalent inertia force considering modal coupling effects*

The equivalent wind loading for the resonant response considering modal coupling effects is given by

\[
\hat{F}_{r_m}(h) = g_{r_m} \Re m \hat{F}_{r_m}(h) \sqrt{1 + \eta_{lm}}, \quad (l = x, y, \theta)
\]

where \(g_{r_m}\) is the peak factor for the \(m\)th resonant response; \(\Re m\) is the RMS modal coordinate neglecting modal coupling effects; \(\hat{F}_{x_m}(h) = \omega_m \sqrt{m_x}(h)\phi_{x_m}(h)\), \(\hat{F}_{y_m}(h) = \omega_m \sqrt{m_y}(h)\phi_{y_m}(h)\), \(\hat{F}_{\theta_m}(h) = \omega_m \sqrt{m_{\theta}}(h)\phi_{\theta_m}(h)\)

is the \(m\)th inertial force corresponding to unit modal coordinate; \(\omega_m\) is the \(m\)th modal circular frequency; \(m_x(h), m_y(h)\) and \(m_{\theta}(h)\) are the structural mass and polar moment of inertia per unit length; \(\phi_{x_m}, \phi_{y_m}\) and \(\phi_{\theta_m}\) are \(m\)th modal shapes. \(\eta_{lm}\) is defined as
The Seventh Asia-Pacific Conference on Wind Engineering, November 8-12, 2009, Taipei, Taiwan

$m$th modal coupling factor given by

\[ \eta_{lm} = \sum_{n=1}^{N} \frac{\phi_{lm}(z)R_m}{r_{nm}} \]

(3)

where \( r_{nm} = \rho_{nm} \text{Re} \left[ \frac{S_{Qn}(\omega)}{\sqrt{S_{Qn}(\omega)S_{Qm}(\omega)}} \right] \); \( \rho_{nm} = \frac{s_{nm}^2(m\zeta_n + \zeta_m)^2}{(1 - \rho_{nm}^2) + 4\zeta_n\zeta_m(1 + \rho_{nm}^2) + 4\zeta_n^2 + \zeta_m^2} \);

\( \text{Re} \left[ S_{Qm}(\omega) \right] \) is cross power spectral density(PSD) between the $n$th and $m$th generalized wind forces; \( S_{Qn}(\omega) \) and \( \zeta_m \) are $m$th generalized wind force PSD and damping ratio, and \( S_{Qm}(\omega) \) and \( \zeta_n \) are the $n$th ones; \( \beta_{nm} = \frac{\zeta_m}{\zeta_n} \).

It is noted that in Eq. (2) \( \sqrt{1 + \eta_{lm}} \) is used to describe contributions of structural mode to the $m$th resonant response. If modal coupling factor \( \eta_{lm} \) is zero, which means the $m$th resonant response is mainly contributed by the $m$th structural mode, Eq. (2) has the same form as the ordinary inertial force equation as presented by Chen(2005).

**Equivalent static wind loading**

Combining LRC method and equivalent inertia force considering modal coupling effects, the total equivalent static wind loading for the peak load effect is written as

\[ \hat{F}_{\ell}(h) = \bar{F}(h) + w_B \hat{F}_{\ellB}(h) + \sum_{m=1}^{N} w_{Rm} \hat{F}_{\ellRm}(h), \quad \ell = x, y, \theta \]

(4)

where \( \bar{F}(h) \) is the mean wind loading; \( \hat{F}_{\ellB}(h) \) is the background wind loading determined by LRC method; \( \hat{F}_{\ellRm}(h) \) is the equivalent wind loading for the $m$th resonant response;

\[ w_B = \frac{s_{\ellB}(h)}{\sqrt{\sum w_{Rm}^2}} \quad \text{and} \quad w_{Rm} = \frac{s_{\ellRm}(h)}{\sqrt{\sum w_{Rm}^2}} \]

are weighting factors; \( \sigma_{\ellB} \) is RMS of background response; \( \sigma_{\ellRm} \) is the RMS of resonant response in mode $m$.

**Case study**

**Wind tunnel test**

A 49-storey concrete building was used to test the proposed framework for 3D ESWL analysis. The building has an overall height of 198m and a floor plan dimension of about 27m by 62m. The fluctuating wind pressures acting on building were obtained from a wind tunnel test. The test was carried out in the Boundary Layer Wind Tunnel of Shantou University whose working section is 3m wide and 2m high. Experiment model with geometric scale of
1:300 is made of organic glass as shown in Fig.1. The typical boundary layer was simulated for suburban terrain category B in accordance with the building’s surroundings. The wind pressures at 430 measuring points were simultaneously measured with sample frequency 300Hz for 36 wind directions shown in Fig.2. In this paper, wind load case in the most unfavorable wind direction of 80-degree was considered. And selected results for the full-scale wind speed of 50 m/s at 10m height are summarized below.

Equivalent static wind loading

Structure FEM analysis results indicate that this building’s mass center and stiffness center are not coincident as shown in Fig.3. The building has the first three fundamental frequencies of 0.20Hz (swaying primary in the Y-direction), 0.28Hz (swaying in the X-direction) and 0.32Hz (torsional mode). As expected, due to the asymmetric structural layout of the building, the mode shape of each of the three fundamental modes is three dimensional as shown in Fig.4.

Fig.5 shows the peak modal inertial loading for the three fundamental modes, with a peak factor of 3.5, which include only resonant components and neglect modal coupling effects. The loads in x and y directions are expressed in terms of load per unit height, whereas the torque is given in terms of torque per unit height.

The weighting factors of the peak modal inertial loads and quasi-static loads for selected response components are summarized in Table 1, which are calculated based on the proposed framework. The response components considered are the displacement in X, Y direction at the top of the building, which are referred to as u, v respectively. Weighting factors of the torsional angle at the building top referred to as θ, base bending moment responses in X and Y direction Mx and My, and the base torque response Mθ are also calculated.

Using the weighting factors, the overall ESWL for peak response can be conveniently determined for immediate applications to building design. Corresponding to the six responses presented above, ESWLs including the quasi-static and the inertial loadings are shown in Fig.6. The peak torsional angle, displacements at the geometric center of the building top storey in X-direction and Y-direction are $0.82 \times 10^{-3}$ rad, 0.0214m and 0.114m respectively, which are calculated by applying ESWLs through simple static structural analysis procedure. By comparison, the three peak responses are also calculated using CQC method which are $0.84 \times 10^{-3}$ rad, 0.0220m and 0.116m. It indicates that responses evaluated using the 3D ESWLs agree well with the results calculated using CQC method.
Fig. 1: Photograph of experiment model

Fig. 2: Definition of wind directions

Fig. 3: Coordinates of centers

Table 1: The weighting factors

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>w₁</td>
<td>0.01</td>
<td>0.645</td>
<td>0.102</td>
<td>0.004</td>
<td>0.636</td>
</tr>
<tr>
<td>w₂</td>
<td>0.75</td>
<td>-0.002</td>
<td>-0.445</td>
<td>0.704</td>
<td>0.001</td>
</tr>
<tr>
<td>w₃</td>
<td>-0.156</td>
<td>0.00</td>
<td>-0.529</td>
<td>-0.23</td>
<td>0.00</td>
</tr>
<tr>
<td>w₄</td>
<td>0.64</td>
<td>0.764</td>
<td>0.715</td>
<td>0.672</td>
<td>0.772</td>
</tr>
</tbody>
</table>

Fig. 4: First three 3D Mode shapes
The analysis of 3D ESWL for a given peak response of buildings with 3D coupled mode shapes of vibration were presented. In this context, the spatiotemporally varying dynamic wind loads may be derived through multiple point synchronous scanning of pressures on the building model surface. Both the cross correlation of wind loads acting in different directions and intermodal coupling among modal response components were taken into consideration in the analysis of response and modeling of ESWL. Using the proposed method, 3D ESWLs of a full-scale 49-storey asymmetric building were calculated. It is found that for a particular response, the wind loadings in all three directions contribute to the response. The peak responses by applying 3D ESWLs are very close with results calculated by CQC methods, which verify the effectiveness of the ESWL evaluation method.

References