Development of Software Package for Design and Analysis of Wind Turbines Blade

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ABSTRACT

In the presented work, development of a GUI software package is described which optimizes the Horizontal axis wind turbine blade geometry in accordance to the local air condition. Mainly BEM theory is used to calculate aerodynamic figures. First all the basic theory and governing equations are discussed in detail. A database of local air properties of different locations of India and a number of NERL S-series airfoil’s aerodynamics database is used. A pseudo code is also included in the appendix. Outcome of the package is a Stereo Lithography*.STL formatted CAD geometry file which can be readily exported to any Modeling package like ANSYS geometry modeler where the rotor blade model can be meshed after minor modification for full CFD validation of the result came from BEM theory.

Keywords: Wind Turbine, Blade design, Design Optimization, Wind Energy, Renewable Energy

Introduction

One of the main factor in the race between Global population and technology advancement is the hunger of energy. Although science is exploring new and improving the existing source of energy like from sea, Nuclear, dams etc. Wind energy will be one of the main feeder.

Research and innovation in material strength and better understanding of wind turbine aerodynamics will keep pushing the upper limit of energy that we can harness from the wind.

In the past ten years India has significantly invested in wind energy production. Influence of Indian Ocean on the average wind speed has been proven to be highly beneficial. Suzlon Group, One of the top wind turbine supplier in the worldand The Ministry of Non-Conventional Energy Sources, Government of India crossed the 1000Mw milestone in 2012 in Jaisalmer wind farm, which shows great potential and government’s will to further reduce the dependency on Non-Renewable sources of energy like coal and bio fuels. Wind mapping of India shows tremendous amount of opportunities [Sharma, A., et al., 2012]. India occupies 5th position in terms of wind energy production after China, US, Germany and Spain and during Jan-Dec 2012, India was the 4th largest country to invest in wind power.
Table 1. Country wise wind energy production by the year 2012
Source: GWEC

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<th>Country</th>
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Mathematical models

A. Wind: Energy in motion

Kinetic energy of a body of mass, $m$ moving with velocity, $v$ is $\frac{1}{2} m v^2$, similarly in the case of wind we can chose a control volume to find the kinetic energy stored in it.

Fig. 2.a Stream tube of air having area $A$ moving with velocity $v$
For a stream tube of area \[ K.E = \frac{1}{2} m v^2 = \frac{1}{2} \rho (Avt) v^2 = \frac{1}{2} Av^3 t \] and power, \( p = \frac{1}{2} Av^3 \)

System (Turbine)

\[ \text{air with velocity, } \vec{v}_1 \rightarrow \text{ System (Turbine)} \rightarrow \text{air with velocity, } \vec{v}_2 \]

Fig. 2.b Wind Turbine Energy flow Diagram

In the fig. 2.b the air with velocity, \( \vec{v}_1 \) interact with the system, S because of which its velocity changes and becomes \( \vec{v}_2 \). If we treat the air and the system as a universe then it is clear that either the air gained some energy or lost some.

Change in energy of air = change in Kinetic Energy = \( p_2 - p_1 = \frac{1}{2} A (v_2 - v_1)^3 \) and it would be beneficial if we can extract this energy and convert it into mechanical energy that in turn can rotate an electric generator shaft or to do some other mechanically heavy work.

B. Wind Turbine as an Energy Extractor

Energy is considered as undefeated perquisite for the human survival on earth. Energy can neither be created nor be destroyed, it only changes from one form to another. Wind Turbine is one of the devices that help us to extract energy from the wind by converting wind’s kinetic energy to turbine’s rotational energy. There are two main variants available. First being HAWT (Horizontal axis wind turbine) and the second VAWT (Vertical axis wind turbine). Due to efficiency and performance, today mostly HAWT is used commercially.
There are four main parts of a wind turbine namely Blades, Tower, Nacelle and Base where components can be categorized as:

- **Mechanical**: Mechanical system consist all the gears and shafts transmission present in nacelle which also houses a generator. The rotational speed is transmitted after speed amplification via multiple set of gears and transmission components to achieve the required rotational speed for power generation.

- **Electrical**: It consists of a generator and controller system which in coordination with anemometer and other sensing devices controls direction rotational speed, pitch control and breaking mechanism.
- Architectural/structural: Wind turbines are one of the huge aerodynamic devices which has to face a variety of environmental conditions, fluttering, blade loading and other forces are so dynamic that if the tuning is not perfect even if the wind is blowing fast we would not be able to get any useful out of it. The most common tower design is about 150-200 feet tall and 10 feet in diameter. When the turbine is in operation it feels rotational effects as well as a large horizontal force because of which the tower must be design such that it do not fails. The base is made of concrete reinforced with steel bars on which the whole tower with nacelle is mounted.

Fig.6 Different composites and materials used in Blade Design

- Aerodynamic: Among the aerodynamics of wind turbine, mechanical, structural and electrical system, Aerodynamics plays the most significant role in the process of energy extraction from the wind. Advancement of computing speed and capabilities enables us to test most of the what-if scenario. Availability of high strength material help us to go for large diameter.

Fig.7 A typical flow streamlines through the rotor
C. Mathematical Modeling of Wind

Earth’s atmosphere is a dynamically changing system and is a function of time, location and distance of point of interest from the surface. For a given location and on the average of time we can easily model a test environment.

a. Wind gradient

As the wind flow over the earth’s surface it experiences shear force in the opposite direction which slows down the air speed near the surface and the formation of boundary layer makes the speed vary as a function of height. Variation of wind speed cause the wind conditions change for a given point on the wind turbine blade as it rotates. Due to lower operation height of wind turbines this effect can significantly affect calculated and field test.

The wind gradient system is analogous to the boundary layer problem in case of flow parallel to a plate but on the account of interaction with human presence such as buildings and other earthy obstacles. We can easily model such case [Banuelos. R.F,C.amacho.C.A and Marcuello.S.R , 2010]. The Monin-Obukhov method is the most widely used to find wind speed at some height in which speed \( v \) and height \( h \) can be related by:

\[
v(h) = \frac{\nu_f}{K} \left[ \ln \frac{z}{z_0} - \xi \left( \frac{z}{L} \right) \right]
\]  

(1)

Where the speed at height \( z \) is \( v \), \( \nu_f \) is the friction velocity, \( K \) is the Karman constant, \( z_0 \) is the surface roughness height, the term \( \xi \left( \frac{z}{L} \right) \) depends upon solar radiation. The Monin-Obukhov method gives good result as compare to experimental data but is difficult to model. One of the methods using Halman constants can reduce the computational time which relates speed \( v \) at height \( h \) by:

\[
\frac{v}{v_o} = (h/h_o)^\alpha
\]

(2)

Where \( h_o \) and \( v_o \) is the reference height and speed, \( \alpha \) is the Halman constant which depends upon the type of location like for ocean, hard and smooth ground the value is 0.10 and for city area with high rise buildings its value is 0.40, this method give good approximations to the actual values.

![Fig.8 Typical velocity gradient profile over a village][Banuelos. R.F,C.amacho.C.A and Marcuello.S.R , 2010]
b. Density and Temperature Gradient

As we go high in air the temperature decreases \( \frac{dT}{dh} \) decreases, \( dp = -\rho g_o dh \) and using ideal gas equation we can find the variation of temperature as:

\[
\frac{\rho}{\rho_0} = e^{-\frac{R T_0 (h-h_0)}{\rho_0}}
\]  

(3)

Where \( \rho_o \) and \( h_0 \) are reference density and height, \( g_o \) is the acceleration due to gravity at the earth’s surface and \( \rho \) is the density at height \( h \).

c. Reynolds number

Flow pattern over a body (Airfoil) depends upon the Reynolds number of the fluid which is defined as \( Re = \rho v l/\mu \) where \( \mu \) is the coefficient of viscosity and it varies with the temperature according to Sutherland’s formula[Smits, A.J. and J.-P. Dussauge, 2006]:

\[
\mu = 1.458 \times 10^{-6} \left[ \frac{T^2}{T + 110.4} \right]
\]

(4)

Where \( T \) is in Kelvin and \( \mu \) is in kg m\(^{-1}\)s\(^{-1}\).

d. Wind Turbine Tower Shed effect

In the presence of wind turbine tower the velocity vector at any point in the plane of turbine disk changes. If we consider any plane parallel to surface and free stream air velocity at some distance from the surface the problem is analogous to the potential flow around a cylinder.

![Fig.9 Streamlines for incompressible potential flow around a cylinder](image)
Consider a 2D cylinder placed in an incompressible and in-viscid flow. Our goal is to find velocity in the plane. The far field velocity is \( \vec{V}_i = U_i \hat{i} + 0 \hat{j} \), where \( U_i \) is constant and equal to far field speed at \( t^{th} \) plane from the bottom. Then the model must satisfy the boundary conditions [Janna, W.S., 1993]

\[
\n\n\n\phi \left( r, \theta \right) = \frac{U_j}{r} \left( r + \frac{R^2}{r^2} \right) \cos \theta 
\]

\[
\n\n\n\dot{v}_r = \frac{\partial \phi}{\partial r} = U \left( 1 - \frac{R^2}{r^2} \right) \cos \theta 
\]

\[
\n\n\n\dot{v}_\theta = \frac{1}{r} \frac{\partial \phi}{\partial \theta} = -U \left( 1 + \frac{R^2}{r^2} \right) \sin \theta 
\]

Although the overall effect on the calculated results will not differ much than if the effect of tower’s shed was neglected and if the computational resources permits we can model this as an optional analysis parameter.

D. Simplest Wind turbine model: Actuator Disc Model

Consider a thin disc of Radius, \( r \) and assuming the flow to be incompressible and non-viscous flow. Using Froude’s Momentum Theory the model can be visualized as:

![Fig.10 Actuator disc model](image)

Herewe can’t apply the Bernoulli’s equation across the rotor plane so let’s divide the region into two. One up to the rotor plane and other from rotor plane onwards.
Bernoulli’s equations in the two regions will be:

Region 1

\[ p_0 + \frac{1}{2} \rho v_0^2 = p_1 + \frac{1}{2} \rho v_d^2 \] (8)

Region 2

\[ p_2 + \frac{1}{2} \rho v_a^2 = p_0 + \frac{1}{2} \rho v_e^2 \] (9)

Thrust acting upon the disc = \( A(p_2 - p_1) = \pi r^2(p_2 - p_1) \) (10)

Using equation (8), (9) and (10) we get \( v_e = v_0 (1 - a) \). Let us define \( a \) as axial induction factor such that wind speed at the disc, \( v_d = v_0 (1 - a) \), i.e. the air speed is reduced by \( v_0 a \) at the disc and further reduced by \( v_0 a \) down the stream, so the downstream far field velocity will become, \( v_e = v_0 (1 - 2a) \).

Similar to axial induction factor, \( a \), we can see that the air approaching the turbine is moving radially away from the central axis and such induction is accounted by the radial induction factor \( a' \). We can note that this induction is much smaller than the axial induction.

Power extracted by the turbine will be the difference between the initial power winds has and the downstream far field wind power. If we neglect the radial induction we can write the power obtained as:

\[
P_{\text{extracted}} = \frac{1}{2} A \rho v_d (v_0^2 - v_e^2) = \frac{1}{2} A \rho v_d^3 4a(1 - a)^2
\] (11)

\[
C_p = 4a(1 - a)^2
\] (12)

Here we can note that the power will be maximum i.e. \( C_p = 0.593 \) when \( a = 1/3 \). In such a model we haven’t considered the induced flow along the blade also the flow distortion at the tip of the blade. Also the model is incapable of providing any information about the blade geometry and other rotor parameter like number of blades used chord and pitch variation along the length of the blade. This model can be anyways used to get a feel about number associated with the sizing and optimal output power.

Not all the energy contained in the wind can be extracted, as shown by the German physicist Albert Betz that no turbine can capture more than 59.3 percent of kinetic energy. Practically only 75% to 80% of this maximum power is possible. [Gorban’ A.N., G.A.M., Silantyev V.M.,2001] Introduced GGS model that considered a non-uniform pressure distribution across the actuator disc which was not considered in the previous model by the Betz, according which the maximum efficiency is much smaller than the efficiency by Betz limit.

Recent research in wind turbines using CFD shows that the actual efficiency is in between the efficiency using GGS model and Betz limit model

E. Mathematical Modeling of Blade:

e. Blade Element Theory
For a rotor, its rotational speed is one of the main key parameter which effects all the properties whether it is power or thrust. Rotational speed can be used to determine all the numbers associated with the rotor. So for a given wind speed let us assume that we have the optimal geometry possible which is unique so the rotational speed.

Consider a \( n \) bladed turbine, for the sake of simplicity we can neglect the aerodynamic interaction then if a property let us take torque, experienced by the central horizontal axis can be calculated by simply multiplying the torque due to one blade by the number of blades.

\[
\pi \theta \rho
\]

Fig. 11 Position of \( i^{th} \) element along the blade

The fig.11 shows the \( i^{th} \) blade element at a distance \( r \) from the central axis having thickness \( dr \). As the blades rotates with rotational speed \( \Omega \), instantaneous speed perpendicular to the length of the blade will be \( \pi r \). Consider a cylinder strip coaxial to the turbine and having radius \( r \) and thickness \( dr \) then the far field airspeed \( v_o \) decreases by a factor \( \lambda_i \), known as axial induction the airspeed at the disk will be \( v_i = (1 - a_i)v_o \) and the rotational induction factor \( a'_i \), \( a_i \), \( a'_i \) and local inflow angle \( \phi_i \) are related as [H., G.,1959]:

\[
a'_i = \frac{1 - 3a_i}{4a_i - 1}
\]

\[
a'_i(1 + a'_i)\lambda_i^2 = a_i(1 - a_i)
\]

\[
\tan \phi_i = \frac{1 - a_i}{\lambda_i(1 + a'_i)}
\]

On solving the first two equations we will have a cubic polynomial in \( a_i \) as:

\[
16a_i^3 - 24a_i^2 + 3(3 - \lambda_i^2)a_i - (1 - \lambda_i^2) = 0
\]

[Maalawi, K.Y., 2001] Shows a trigonometric transformation approach to solve the three degree equation (16) with transformation \( a = b \cos \theta + 1/2 \) and comparing with the matrix identity \( 4\cos^3 \theta - 3\cos \theta = \cos 3\theta \) the solution that give positive value of roots in the permissible range is

\[
a = \frac{1}{2}[1 - \Lambda_r(\cos \theta^+ - \cos \theta^-)]
\]

Where \( \theta^\pm = \frac{1}{3}\cos^{-1}(\pm \Lambda_r^{-1}) \) and \( \Lambda_r = \sqrt{1 + \lambda_i^2} \)
But we as we know that the value of $\phi$ is in between 0.25 to 3.5 we can use an iterative method to find the root by setting the initial value of $\phi$ to 0.2 and slowly increasing it till we get the desired accurate root.

Fitting downwash angle and speed ratio obtained from Glauert optimal model into five degree polynomial we can obtain downwash angle at any speed ratio, similar work done in [Nathan GK.,1980] obtained

$$\phi = 57.51 - 35.56 \lambda_i + 10.61 \lambda_i^2 - 1.586 \lambda_i^3 + 0.114 \lambda_i^4 - 0.00313 \lambda_i^5$$  \hspace{1cm} (19)

Where $\lambda_i = \omega r/v_i$ is defined as the speed ratio for the $i^{th}$ blade element. The effective angle of attack will be $\alpha_i = \phi_i - \sigma_i$, where $\sigma_i$ is equal to the section setting angle.

Using Equation 13, 14, 15, 16 and 19th we can design an iterative method to find the angular speed and once we get the angular speed we can start calculating the axial and radial induction factor along the length of the blade.

For a given airfoil the optimal angle of attack and the foil setting angle depends upon the Reynolds number. So for s section at a distance $x$ from the central axis first we need to calculate the effective airspeed and the corresponding Reynolds number

The relative airspeed will be $V_r = \sqrt{[\Omega r(1 + a')^2 + [v_i(1 - a)]^2}$ at this point we don’t know the chord length of the section foil so we can use a tabulated data 5.6 of chord variation at the non-dimensional length given in[Wilson, R.E. and D.A. Spera,2003] p.296, this will serve us a decent and logical initial guess for the chord. We can repeat all the steps from this point by setting the chord angle obtained to the guess value such that a particular level of accuracy is reached.

Optimal lift and drag coefficient at corresponding optimal angle of attack can be calculated by:

$$C_l = \sqrt{C_{D_0}/K_2}$$  \hspace{1cm} (20)

$$C_d = 2C_{D_0} + K_1 C_l$$  \hspace{1cm} (21)
Where $K_1$ and $K_2$ are the solution of the following matrix equation

$$
\begin{bmatrix}
C_{D_n} - C_{D_0} \\
C_{D_m} - C_{D_0}
\end{bmatrix} =
\begin{bmatrix}
C_{l_n} & C_{l_n}^2 \\
C_{l_m} & C_{l_m}^2
\end{bmatrix}
\begin{bmatrix}
K_1 \\
K_2
\end{bmatrix}
$$

(23)

Where $n \neq m$ and $C_{D_0}$ is the drag coefficient at zero angle of attack.

If $C_l$ and $C_d$ denotes lift and drag coefficient at a particular angle of attack then for a $n$ bladed rotor system the thrust and drag forces experienced by corresponding elements in each blade will be:

$$
\text{Thrust}, T_i = \frac{1}{2} \rho_i v_i^2 n c_i dr (C_l \cos \phi + C_d \sin \phi)
$$

(24)

$$
\text{Torque}, Q_i = \frac{1}{2} \rho_i v_i^2 n c_i r dr (C_l \sin \phi - C_d \cos \phi)
$$

(25)

Using Actuator Disk Theory for a given stream tube coaxial to the rotor as shown in the fig.13, we get

$$
thrust, T_i = \Delta KE = 4 \rho_i \pi r v_i^2 a_i (1 - a_i) dr
$$

(26)

$$
\text{Torque}, Q_i = 4 \rho_i \pi r^3 v_i \Omega a_i' (1 - a_i) dr
$$

(27)

Fig.13 Stream tubes of actuator Disc model

Equating both the equations of thrust and torque we can find the optimum chord length, $c$ required for the cross section by the following two expressions and the Normal force coefficient $C_N$

$$
\frac{a_i}{1 - a_i} = \frac{C_N n c \cos \phi}{8 \pi r \sin^2 \phi}
$$

(28)
\[
\frac{a_i'}{1 + a_i'} = \frac{C_N n c}{8 \pi r \cos \phi}
\]

(29)

And \( C_N = C_{ti} \cos \phi + C_{di} \sin \phi \)

(30)

Using blade element theory \( C_{Ti} = \frac{T_i}{\frac{1}{2} \rho_i v_i^2 c d r} \), and \( C_{Ti} = 4 a_i (1 - a_i) \), by momentum theory is known as thrust coefficient can be calculated by a simple third degree empirical relation as.[Maalawi, K.Y., 2001]:

\[
a_i = 0.27 C_{Ti} + 0.10 C_{Ti}^3
\]

(31)

Which can be used for cross checking or as a convergence criteria.[Nathan GK., 1980] Shows the fact that \( 0.96 < C_{Ti} < 2.0 \)

\section*{f. Approximations:}

\subsection*{(i). Tip loss modeling:}

Up-to this point we haven’t considered the effect of flow along the length and the tip loss. Prandtle’s tip-loss factor,

\[
F(x) = \frac{2}{\pi} \cos^{-1} \left( e^{-\frac{n}{2} \frac{R-x}{r \sin \phi}} \right)
\]

(32)

Can be used to modify the force coefficient tangential velocity of the element to the air as:

\[
C_{Ti} = 4 a_i F(1 - a_i) \quad \text{if} \quad a_i \leq a_c
\]

(33)

\[
C_{Ti} = 4 a_i F [a_c^2 + (1 - 2a_c) a_i] \quad \text{if} \quad a_i \geq a_c
\]

(34)

Where \( a_c \geq \frac{1}{3} \) in order that Betz limit is not exceeded. Betz’s law is used to calculate the maximum power that can be extracted from the wind, independent of blade geometry and design[Spéra, D.A., 1995]. It assumes that the flow is coaxial, incompressible and the rotor is having infinite number of blades

\subsection*{(ii). Unnoticed Effects:}

Due to unforeseen chain of events, Cascade effects can come in the play. Cascade effects are usually seen in a tree structures, known as tree events. In context to wind turbine aerodynamics we can take the effect of distortion of flow field due to finite thickness and width.

Due to change in flow field, effective angle of attack changes thus the lift coefficient. One can modify the existing angle of attack calculated up to this point. At a distance \( x \) from the center, angle of attack for the corresponding airfoil section can be written as
\[
\alpha_i = \phi_i - \sigma_i + \varepsilon_i \tag{35}
\]

Where \( \varepsilon_i = \varepsilon_1, \) due to finite thickness + \( \varepsilon_2, \) due to finite width.

After interaction from one blade the axial speed velocity increases. In fig. 14 the airfoil sections are shown closer for the sake of illustration and the same distance is very large so we can analyze the system as a 2D problem.

On applying continuity equations the width \( w_i, \) distortion gap created due to the presence of second airfoil in terms of local thickness, \( t \) normal to chord line satisfy the relation.[Dugundji, J., E. E. Larrabee, and P. H. Bauer., 1978]:

\[
dw = \frac{B \phi_i}{2\pi r} \int_0^c t \, dz \tag{36}
\]

The flow displacement increases the angle of attack by \( \varepsilon_1 = w/c \)

\[
\varepsilon_1 = \frac{B \phi_i}{2\pi rc} \int_0^c t \, dz \tag{37}
\]

Where \( z \) is the chord wise coordinate from the leading edge.

The value of induces tangential velocity changes from 0 at leading edge to \( 2a'_i \Omega r \) but due to finite blade width the circulation developed by the blade changes because of which the effective angle of attack changes by:

\[
\varepsilon_2 = \frac{1}{4} \left\{ \tan^{-1} \left[ \frac{(1 - a_i)z}{(1 + 2a'_i)} \right] - \tan^{-1}[(1 - a_i)z] \right\} \tag{38}
\]

In general the twist of an airfoil section is done at \( z = c / 4 \) of chord length.
F. An Overview of Database Used

In India the density of air varies not only with location but also with the time. From the Fig. 15 we can see the fluctuation in the density. The power output of a wind turbine can be approximately related with the $P \propto \rho$ relation keeping all the parameter constant so more the density more the power we can obtain for a given blade geometry. Also we can note that the air density in Trivandrum is nearly constant while in Gauhati it fluctuates. In such case stating whether installing same types of turbine at these location which will produce more energy becomes difficult.

![Fig.15 Variation of Density over the months at different Locations](image1)

![Fig.16 Variation of temperature over the month at different locations](image2)
Air speed is the main and the most important player in the wind turbine aerodynamics. Fig. 17 shows its variation over the months. Here we can see that the wind speed reaches maximum during the month of June for Trivandrum which is also almost 2 times than the others while the lower bound is almost same for all the location. As the optimal blade geometry is heightly dependent on the airspeed the blade designed for one place need not to be the perfect blade for the other locations. So we must see the capability of each blade for each location before installing it. Also the airspeed fluctuations over the month make every blade design somewhat not the optimal what it can be.

Fig. 17 Wind Speed Variation over the month at different locations

Table 2. Available NREL S-Series Airfoils, their usage category and operating reynolds numbers

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</tbody>
</table>
As the blade rotates the relative wind speed is a function of its rotational speed so the Reynolds number. It forces us to use different airfoils at different location of the blade to get the optimal power. Over the year the National Renewable Energy Laboratory (NREL) has researched on the various airfoil that can be used for the wind turbines blade and compiled a large number of airfoils specific to blade location like root, tip and main. The table 2 contains some of the root, tip and main airfoils having different operating Reynolds number.

![Fig.18 Cl vs AOA](image)

**Application (A case study)**

User Input:

- Inner Radius = 2
- Outer Radius = 50
- Wind Speed = 8 ms⁻¹

\[
\text{angular speed} = \text{find_angular_speed}() \\
\text{initial guesses} = [15, 16, 17, ..., 25] \\
\text{ans} = 14.4225 \text{ RPM} \\
\text{optimal twist} = \text{find_optimal_twist}() \\
\text{for airfoil ID} = 'S816'
\]

\[
[K_1, K_2] = [-0.0074, 0.0135] \\
C_L = 0.7200 \\
C_D = 0.008672
\]
Fig. 19 Optimal twist variation

Fig. 20 Optimal chord variation

Power Output = 513.887 KW

Fig. 21 Main GUI
Conclusions

Discussion and analysis of different air properties at different locations in India shows that it is not guaranteed that the wind turbine designed for a location will also perform at its best at the other locations round the year because of which there is need of having different design parameters for different location and the presented tool can be a good analysis tool for approximate insight into the potential for wind power generation for a given location. Having computationally less expansive methods allow us to calculate total output power for an interval of one month, one year or for the whole time of operation. Insiste of having rotor’s rotational speed as an user input the program is capable of finding the rotational speed using an iterative approach which also eliminates the possibility of getting negative, complex or out of range axial induction factor. The program is also capable of finding the best suitable airfoil from a set of airfoils predefined in the database to get maximum power output. Presently only the aerodynamically optimal geometry is being calculated and will also consider structural and other factors in future studies. Once we get the desirable power we can easily confirm it by full CFD analysis using inbuilt geometry export function in the application.

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1197
Appendix: Pseudo code

(a) Class Wind

Properties
Air_Speed
Density
Kinematic_Viscosity

Methods
Construct_sample_wind()

mesh : height , h
wind gradient , \( v = v_o(h/h_o)^\alpha \)
get : \( \frac{dT}{dh} = a \)
temperature , \( T = T_0 - ah \)
density variation , \( \rho = \rho_o e^{-\frac{[\rho_o(h-h_o)]}{KT}} \)
kinematic viscosity , \( \mu = 1.458 \times 10^{-6}\frac{3}{(T+110.4)} \)

save : \( v, \rho \) and \( \mu \)

(b) Class Rotor

properties
rotational_speed

methods
Find_Angular_speed()
initialize Angular_Speed , \( \Omega = 1 \) rpm
while tollerence < 0.05 rpm

do

Take any \( i^{th} \) element at distance \( r \) \{ (0.3R, 0.7R) \)
Find : speed_ratio, \( \lambda_i = \Omega r / v_i \)
axial_induction, \( a_i = \text{solve} (16a^3_i - 24a^2_i + 3(3 - \lambda_i^2)a_i - (1 - \lambda_i^2) = 0) \)
radial_induction , \( a' = \frac{1-3a}{4a-1} \)
Downwash_angle \( \phi = 57.51 - 35.56 \lambda_i + 10.61 \lambda_i^2 - 1.586 \lambda_i^3 + 0.114 \lambda_i^4 - 0.00313 \lambda_i^5 \)
new_angular_speed = \( \frac{(1-a_i)v}{r(1+a_i')\tan \phi} \)
tollerence = new_angular_speed - Angular_speed
set : Angular_Speed = new_angular_speed

save : Angular_Speed\( \Omega \)
(c) Class Airfoil

Properties

Chord
Twist

Methods

Find_downwash_angle ( )

For $i^{th}$ element

$\Omega =$ Angular_Speed

Find : speed_ratio, $\lambda_i = \Omega r / v_i$

Downwash_angle $\phi = 57.51 - 35.56 \lambda_i + 10.61 \lambda_i^2 - 1.586 \lambda_i^3 + 0.114 \lambda_i^4 - 0.00313 \lambda_i^5$

axial_induction, $a_i = \text{solve} \left( 16 \alpha_i^3 - 24 \alpha_i^2 + 3 \left(3 - \lambda_i^2\right) a_i - \left(1 - \lambda_i^2\right) = 0 \right)$

radial_induction, $a' = \frac{1 - 3 \alpha}{4 \alpha - 1}$

save $\phi, a_i$ and $a'$

Find_Optimal_twist ( )

for $i^{th}$ element rotating with speed,

$V_r = \sqrt{[\Omega r(1 + a')]^2 + [v_i(1 - a)]^2}$

$Re = \text{Call: find_reynolds_number ( )}$

$Re = \rho v l / \mu = v l / \nu$

$[ cl , cd ] = \text{read : airfoil_data for reynolds number , Re}$

solve for $K_1$ and $K_2$ using any method like Jacobi and Gauss-Seidel Method

$\begin{bmatrix} C_{D_n} - C_{D_0} \\ C_{D_m} - C_{D_0} \end{bmatrix} = \begin{bmatrix} C_{t_n} & C_{t_n}^2 \\ C_{t_m} & C_{t_m}^2 \end{bmatrix} \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$

$n \neq m$

save : $C_t = \sqrt{C_{D_0} / K_2}$

save : $C_d = 2C_{D_0} + K_1 C_t$

optimal_angle_of_attack , $\alpha = \frac{c_t}{d c_t / d \alpha} + \alpha_0$

optimal_twist, $\sigma = \phi - \alpha$

Find_optimal_chord ( )

1199
For i\textsuperscript{th} element

Tangential_aerodynamic_coefficient, $C_T = C_i \sin \phi - C_d \cos \phi$

Normal_aerodynamic_coefficient, $C_N = C_i \cos \phi + C_d \sin \phi$

little_f = \frac{\text{Num. of blades} (R-r)}{2 r \sin \phi}$

prandtl's_tip_loss_factor, $F = \frac{2}{n} \cos^{-1}(e^{-\text{little}_f})$

Modify_C_T ( )

Using Betz's law, $a_c = 1/3$

if $a_i \leq a_c$

$C_T = 4a_iF(1 - a_i)$

if $a_i \geq a_c$

$C_T = 4a_iF[a_c^2 + (1 - 2a_c)a_i]$

local_solidity, $\sigma = 4F \frac{a}{(1-a)} \sin^2 \phi$

save :$C_T, C_N$ and optimum_chord, $C = \frac{2\pi \sigma}{\text{Num. of blades}}$

Find_physical_twist ( )

For i\textsuperscript{th} element

error_due_to_finite_thickness ( )

mesh airfoil's area, $aF_j = \left[ \begin{array}{l}
\text{length} = t_j \\
\text{breadth} = z_{j+1} - z_j
\end{array} \right]$

$\varepsilon_1 = \frac{B \phi_i}{2\pi c} \sum_{j=1}^{\text{num. of arial grids}} aF_j$

error_due_to_finite_width ( )

Distance of point of twist from leading edge, $z = C / 4$

$\varepsilon_2 = \frac{1}{4} \left\{ \tan^{-1} \left[ \frac{(1-a_i)z}{(1+2a_i)} \right] - \tan^{-1} \left[ (1-a_i)z \right] \right\}$

$\sigma_{\text{physical}} = \sigma_{\text{aerodynamic}} + \varepsilon_1 + \varepsilon_2$

save :$\sigma_{\text{physical}}$

Find_aerodynamic_forces ( )

For i\textsuperscript{th} element

grid_area, $gA = C_i (r_{i+1} - r)$
save : elemental_thrust, \( T_i = \frac{1}{2} \rho v_i^2 C_N \) gA

save : elemental_torque, \( Q_i = \frac{1}{2} \rho v_i^2 r C_T \) gA

save : elemental_power, \( P_i = \frac{1}{2} \rho v_i^2 r C_T \) gAΩ

(\textit{d}) Class \textit{Wind\textunderscore Turbine}

properties

thrust

power

CAD\textunderscore modal

methods

Mesh\textunderscore along\textunderscore Blade\textunderscore Legth ( )

\[ gB_i = \begin{bmatrix}
\text{length} = C_i \\
\text{thickness} = r_{i+1} - r
\end{bmatrix} \]

find\textunderscore power\textunderscore and\textunderscore thrust ( )

Power , P = 0

Thrust, T = 0

sample\textunderscore air = call : wind ( )

for \( i = 1: \text{num\textunderscore of\textunderscore grids} \)

do

\begin{align*}
\text{rotational\textunderscore speed} &= \text{call : blade} \\
\text{x} &= x_i \\
[P_i, T_i] &= \text{call : airfoil} \\
\text{P} &= \text{P} + P_i \\
\text{T} &= \text{T} + T_i \\
\text{save : twist\textunderscore at\textunderscore r, } &\alpha_i = \alpha_{\text{physical}} \\
\text{save : chord\textunderscore at\textunderscore r, } &C_i = \text{optimal\textunderscore chord} , C
\end{align*}

save : Power, P

save : Thrust, T

Generate\textunderscore CAD ( )

[ \text{x\textunderscore array, y\textunderscore array} ] = \text{read : airfoil\textunderscore coordinates}

for \( i = 1 : \text{num\textunderscore of\textunderscore grids} \)

do
rotate about c /4 ( )
  u = x_array – C /4
  v = y_array

rotated_complex_vector, z = (u + iv) e^{i(twist at r)}
save : new_x_array_at_r = real_part_of_complex ( z ) + C / 4
save : new_y_array_at_r = imaginary_part_of_complex ( z )
save : new_z_array_at_r = r

generate_STL_formated_CAD ( )
  create ASCII file : 'geometry.STL'
  write : "solid(name"
  N = find : Number_of_triangles_possible ( )
  for i = 1 : N
      do
          [(x1, y1, z1), (x2, y2, z2), (x3, y3, z3)] = save temporarily : i^{th} triangle
          find_unit_normal ( )
          [\vec{t}_1, \vec{j}_1, \vec{k}_1] = [x_2 – x_1, y_2 – y_1, z_2 – z_1]
          [\vec{t}_2, \vec{j}_2, \vec{k}_1] = [x_3 – x_1, y_3 – y_1, z_3 – z_1]
          [\vec{n}_i, \vec{n}_j, \vec{n}_k] = cross_product \{[\vec{t}_1, \vec{j}_1, \vec{k}_1], [\vec{t}_2, \vec{j}_2, \vec{k}_1]\}
          Save temporarily : \hat{\vec{n}}_i, \hat{\vec{n}}_j, \hat{\vec{n}}_k = \frac{[\vec{n}_i, \vec{n}_j, \vec{n}_k]}{modulus [\vec{n}_i, \vec{n}_j, \vec{n}_k]}
          writing_file ( )
          write - append : string = "facet normal \hat{\vec{n}}_i \hat{\vec{n}}_j \hat{\vec{n}}_k"
          outer loop
          vertex x_1 y_1 z_1
          vertex x_2 y_2 z_2
          vertex x_3 y_3 z_3
          endloop
          endfacet"
      clear temporary memory
      write - append : "endsolid name"
  close File : 'geometry.STL'
User Input

Get : Location = “XYZ”
Get : Hub_Radius = R_1
Get : Rotor_radius = R_2

Sample_Air = Call : Wind
Optimal_Rotational_speed_of_the_Rotor = Call : Rotor

Find : N , Number_of_Airfoil_in_Database

For i = 1 : N

do

find : Power ( i ) = Find_power ( )

Temperary_Airfoil = get : Airfoil_ID ( i )
Temperary_Airfoil_data = Call : Airfoil ( Temperary_Airfoil)
Power ( i ) = Call : Wind_turbine ( Temperary_Airfoil_data )

[ Maximum_power_extractable , Airfoil_ID ] = [ find : max { Power ( i ) } , i = K ]

[ Max_Power , Thrust , CAD ] = Final_results ( K )

Airfoil_data = Call : Airfoil (Airfoil_ID)

[Power , Thrust , CAD ] = Call : Wind_turbine (Airfoil_data )