ESWL for Large-span Roof Based on Grouping Response Method
Xuanyi Zhou¹, Ming Gu², Gang Li³

¹Associate professor of State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, Shanghai, China, zhouxytj@tongji.edu.cn
²Professor of State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, Shanghai, China, minggu@tongji.edu.cn
³Engineer of State Key Laboratory of Disaster Reduction in Civil Engineering, Tongji University, Shanghai, China, freeskylg@126.com

Abstract: The method for equivalent static wind loads applicable to multi-responses is proposed in this paper. A modified load-response-correlation (LRC) method corresponding to a particular peak response is presented, and the similarity algorithm implemented to the group response is described. The main idea of the algorithm is that two responses can be put into one group if the value of one response is close to that of the other response, when the structure is subjected to equivalent static wind loads aiming at the other response. Based on the modified LRC, the grouping response method is put forward to construct equivalent static wind loading. This technique can simultaneously reproduce peak responses for some grouped responses.

Key words: large-span roof; equivalent static wind loads; modified LRC method; grouping response; similarity algorithm

Introduction
An alternative approach is to present wind loading data in the form of equivalent static wind loading (hereafter referred to as ESWL), which produces peak responses. From the beginning, researchers adopted a simplified approach to represent ESWL involving the calculation of a “gust response factor” (Davenport, 1967; Ueda, 1994; Uematsu, 2002, 2008). The implication of this approach is that the equivalent static wind load distribution corresponding to the maximum response has the same shape as the mean wind load distribution. For some roofs with specific geometry, the “gust response factor” method is applicable. Empirical formulas were proposed by Uematsu et al. (2002, 2008) to calculate load distributions on several kinds of structures based on the characteristics of dynamic responses. The load-response-correlation (LRC) method (Kasperski, 1992) is regarded as an important milestone in research development. Furthermore, for large-span roofs, a new method composed of background equivalent static wind loads from the LRC method and inertial wind loads was recently proposed by Holmes (2002) to compute the equivalent static wind loads on roofs. To improve the method of Holmes (2002), Gu and Zhou (2009) adopted the modal coupling factor, which can quantitatively describe the contribution of modal coupling effects to the resonant response. Further, Zhou and Gu (2010) applied the method to calculate the equivalent static wind loads for large-span roof structures. A characteristic of the current methods for ESWL is that they only aim at determining a specific response. To compensate for this deficiency, Katsumura (2007) brought forward a method to compute the ESWL that can simultaneously reproduce peak responses in all structural members. A new equivalent static wind distribution was named universal ESWL distribution by Katsumura.

When the method for reproducing multi-responses is applied to complicated roof structures, large deviations from natural wind pressure distribution may be found in the universal ESWL distribution. To simultaneously satisfy all peak responses, some erratic ESWL distributions are obtained and the values of equivalent static wind pressure could reach tens or, in some instances, hundreds of KPa. In this paper, the rationality of ESWL distribution is judged by its similarity to natural wind pressure distribution, especially the magnitude of wind pressure. Hence, the goal of ESWL presented in this paper is to reproduce a certain group of peak
responses instead of all structural peak responses. To obtain the reasonable equivalent static wind distribution, a modified LRC method corresponding to a particular peak response is first put forward in the paper. The similarity algorithm explaining the process of grouping response is described. Based on the modified LRC, the grouping response method is then proposed to compute ESWL, which can simultaneously reproduce peak responses for some grouped responses.

1 Method for Calculating Equivalent Static Wind Loading

1.1 Modified LRC Method

The LRC method of Kasperski (1992) has been used to determine the background component of equivalent static loading. The LRC method can give an expected equivalent load distribution corresponding to a particular load effect or response $i$, with an influence coefficient vector $I_i$. The equivalent wind loading is given by

$$P_{ei} = \rho_p \sigma_p \frac{I_i^\top}{\sigma_{ri,B}}$$

where the subscript ‘B’ denotes background component; $\rho_p$ is the correlation coefficient between fluctuating wind loading and background response; $\sigma_p$ is the RMS of wind pressure; $\sigma_p^2$ represents the covariance matrix of fluctuating wind loading; and $\sigma_{ri,B}$ is the RMS of background component of response $i$.

Generally, the LRC method is used only to calculate the background component of equivalent static load distributions; at the same time, many other methods can be chosen to compute the resonant component. To circumvent the process of calculating the resonant component, the modified LRC method, which regards a specific total peak response $\hat{R}_i$ as its direct target, is presented below. The peak response $\hat{R}_i$ is defined by

$$\hat{R}_i = \bar{R}_i \pm g \sigma_{ri,i}$$

where $\bar{R}_i$ is the mean response; $\sigma_{ri,i}$ represents the RMS response; and $g$ is the peak factor, set here as 2.5. The sign “±” assures that $\hat{R}_i$ obtains the absolute maximum.

In the modified LRC method, the form of equivalent static loading obtained from the traditional LRC method is preserved. For the sake of considering the resonant component, each original element in the background component $P_{ei,B}$ is multiplied by a modified coefficient. Thus, the equivalent wind loading in the modified LRC method is given by

$$P_{ei} = \bar{P} + \text{diag} (P_{ei,B}) k_{eBi}$$

where $\bar{P}$ is the vector of mean wind loading; $\text{diag}(\cdot)$ denotes that a vector is expanded to a diagonal matrix; the second term on the right side of the equation is the dynamic component in which $P_{ei,B}$ is the background component calculated from the traditional LRC method; $k_{eBi}$ is the unknown modified coefficient vector used to compensate the background component for the resonant component. The variable $k_{eBi}$ can be computed as follows:

$$I_i P_{ei} = I_i (\bar{P} + \text{diag} (P_{ei,B}) k_{eBi}) = \hat{R}_i$$

Eq. (4) can be rearranged as

$$k_{eBi} = \left[ I_i \cdot \text{diag} (P_{ei,B}) \right]^\top \left( \hat{R}_i - I_i \bar{P} \right)$$

where $[ \cdot ]^\top$ denotes the Moore-Penrose generalized inverse (Golub, 1996). The results of $k_{eBi}$ obtained through Eq. (5) is the minimum norm solution (Bhatia, 1997) for Eq. (4).
Substituting $k_{eij}$ into Eq. (3) leads to the equivalent static loading corresponding to the peak response $R_i$. Similar to the traditional LRC method, the equivalent static loading computed through the modified LRC method only aims at a specific response.

1.2 Similarity Algorithm

If the calculating error of one response when the structure is subjected to the equivalent static wind loads aiming at other response is small, two responses could be put into the same group. The similarity algorithm that goes into the computer implementation is described below.

Step 1. Equivalent static loading $P_{ei}$ corresponding to each response $i$ is calculated using the modified LRC method. A matrix of equivalent static loading $P_e$ can be constructed, which consists of vectors of equivalent static loadings $P_{ei}$. The matrix of equivalent static loading is denoted as $P_e = [P_{e1}, P_{e2}, \ldots, P_{en}]$, where $n$ is the number of all targeted responses.

Step 2. The matrix of influence coefficients is set as $I, I = [I_1, I_2, \ldots, I_n]^T$. Using $P_e$ and $I$, the matrix of responses $R$, corresponding to all equivalent static loadings can be achieved.

$$R = IP_e = \begin{bmatrix} R_{s11} & R_{s12} & \cdots & R_{s1n} \\ R_{s21} & R_{s22} & \cdots & R_{s2n} \\ \vdots & \vdots & & \vdots \\ R_{sn1} & R_{sn2} & \cdots & R_{snn} \end{bmatrix}$$

where $R_{sj}$ is the value of response $i$ when the structure is subjected to the equivalent static loading corresponding to response $j$.

Step 3. The matrix of computational accuracy is obtained through dividing each line of $R$ by the diagonal element of line. The element in $T$ is $T_{ij} = R_{ij}/R_{ii}$, which defines the computational accuracy for response $j$ when the structure is subjected to the equivalent static loading corresponding to response $i$. The variable $T_{ij}$ is generally less than 1.0 while $i \neq j$; $T_{ij}$ is equal to 1.0 when $i = j$, which shows that the error is zero.

Step 4. At the final step, the value of similarity level $\alpha$ should be evaluated. If $T_{ij}$ is larger than $\alpha$, the responses $i$ and $j$ can be put into one group. The value of $\alpha$ could differ depending on the structure. If a high level of similarity is expected, the $\alpha$ value can be large. Practical experience shows that similarity between responses in the same group can be guaranteed when $0.5 \leq \alpha$. When the criterion of $\alpha \geq 0.5$ is chosen, it does not mean that, under the ESWL presented in Section 1.3, the responses could be estimated with an error of as large as 50%. The four steps above are only to group similar responses.

1.3 Equivalent Static Wind Load Distribution for Grouped Responses

Responses can be grouped based on the modified LRC method and similarity algorithm. Due to similar characteristics of responses in the same group, equivalent static loading for a group can be expressed as a linear combination of the equivalent static loadings for every response in the group; that is,

$$P_{e,M} = \sum_{i=1}^{m} k_{Mei} \cdot P_{ei} = P_{Me} \cdot k_{Me}$$

(6)
where \( P_{e,M} \) is the equivalent static loading for the \( M^{th} \) group; and \( P_{ei} \) is the equivalent static loading corresponding to the peak response \( i \), in the \( M^{th} \) group, calculated using the modified LRC method. The element \( k_{Me} \) in vector \( k_{Me} \) is the weighing factor that indicates the contribution from the equivalent static loading aiming at response \( i \), to the equivalent static loadings for the group. The matrix of equivalent static loading \( P_{Me} = [P_{e1}, P_{e2}, \ldots, P_{em}] \), is \( P_{Me} \).

The number of responses in the \( M^{th} \) group is \( m \). According to the requirement of equivalent static loading, \( k_{Me} \) must ensure the minimum total error of peak responses when equivalent static loading is applied to the structure. Therefore, \( k_{Me} \) should satisfy the equation below

\[
\min_{k_{Me}} \sum_{i=1}^{m} (I_M P_{Me} k_{Me} - \hat{R_i})^2
\]

Let \( I_M = [I_1, I_2, \ldots, I_m]^T \); then \( I_M \) the matrix of influence coefficient of the \( M^{th} \) group. Let \( \hat{R}_M = \{\hat{R}_1, \hat{R}_2, \ldots, \hat{R}_m\}^T \); \( \hat{R}_M \) is the peak response vector of the \( M^{th} \) group. Then, Eq. (7) can be written as

\[
\min_{k_{Me}} \|I_M P_{Me} k_{Me} - \hat{R}_M\|_2
\]

where \( \| \cdot \|_2 \) denotes the Euclid norm of vector. Eq. (8) is a linear least-square problem (Ipsen, 2009), and \( k_{Me} \) can be obtained by

\[
k_{Me} = [I_M P_{Me}]^+ \hat{R}_M
\]

where \([\cdot]^+\) denotes the Moore-Penrose generalized inverse as mentioned above. Substituting \( k_{Me} \) into Eq. (6) leads to equivalent static loadings corresponding to all peak responses in the \( M^{th} \) group.

The linear least-square problem above [Eq. (8)] is related to the contradictory equation as presented below:

\[
I_M P_{Me} k_{Me} = \hat{R}_M
\]

If the rank of matrix \([I_M P_{Me}, \hat{R}_M]\) is equal to that of \([I_M P_{Me}, \hat{R}_M]\), Eq. (9) is the unique solution to Eq. (10); if the rank of matrix \([I_M P_{Me}, \hat{R}_M]\) is not equal to that of \([I_M P_{Me}, \hat{R}_M]\), Eq. (9) is the least-squares solution or minimum norm solution to Eq. (10) (Bhatia, 1997).

From the description above, the equivalent static wind loading for a group proposed in the paper is actually a linear combination of individual ESWLs for the grouped responses. For these individual ESWLs are not orthogonal and complete basis set, the obtained ESWLs is just one of many possible solutions by using the Moor-Penrose generalized inverse. Some work should be done in the future research to make up for the weakness. On the other side, a meaningful distribution of ESWL could be still obtained through the current method in the paper for the basic component of the ESWLs originates from the LRC method. From the application of the method in section 3, it can be found that, when the appropriate groups are chosen, the range of magnitude of equivalent static wind loads is similar to that of natural wind pressure, which is the “rationality” of ESWL distribution mentioned in the section of Introduction.
2 Wind Tunnel Experiment and Characteristics of Wind Pressure Distribution

2.1 Wind Tunnel Experiment

The application of the method to actual large-span roof structures is investigated. The large-span roof set used as an example is a steel cylindrical shell with double-layer space grids. The structure has a span of 103.0 m, a height of 40.0 m, and a length of 140.0 m. Its rise-span ratio is 0.39. To obtain pressure distribution on the roof surface, a wind tunnel test was performed in the TJ-2 Boundary Layer Wind Tunnel at Tongji University, whose working section is 3.0 m wide and 2.5 m high. The geometry scale was 1:150. A total of 430 measuring taps were arranged on both the top and bottom of the roof in a grid pattern, meaning there were 215 measuring points. Fluctuating wind pressures were simultaneously measured at all 430 measuring taps on the rigid model of the roof at 300 Hz. Fig. 1 shows the parameters of roof and wind directions. Pressure taps were connected with the measurement system through PVC tubing. To avoid dynamic pressure distortion, signals were modified using the transfer function of the tubing systems.

The wind field of terrain category B in accordance with the Chinese Code (2006) was simulated with a standard spire-roughness arrangement on the wind tunnel floor. The exponent of the mean wind speed profile for terrain category B was 0.16. Reference wind speed at the height of 1.0 m (equivalent to 150.0 m high in the atmospheric boundary layer) in the wind tunnel for the measurement obtained from the pitot tube was 12.0 m/s, indicating that the velocity scale was 1:4.85. Turbulence intensity at the height of the roof top was about 15%. Fig. 2 presents the power spectra at the height of the roof top in the wind tunnel.

Wind pressure obtained from the pitot tube was used to calculate the non-dimensional pressure coefficients. Non-dimensional net pressure coefficient \( C_p \) is defined by

\[
C_p = \frac{P_{u,k} - P_{b,k}}{(P_o - P_\infty) \times \left(\frac{40}{150}\right)^{0.32}}
\]

where \( C_{p,k} \) is the pressure coefficient at the \( k \)th measuring point; \( P_{u,k} \) is the upper pressure at the \( k \)th measuring point; \( P_{b,k} \) is the corresponding bottom pressure; and \( P_\infty \) and \( P_o \) are the static pressure and total pressure of the pitot tube at the reference point in the test, respectively. Wind speed at the height of the roof top (40.0 m) was used to obtain the normalized pressure coefficient.

2.2 Brief Description of Characteristics of Wind Pressure Distributions

Only a brief analysis on the wind pressure at a 90° wind direction is given below. The method of equivalent static wind loads is also applied only at the 90° wind direction in the next section. Fig. 3(a) shows the contours of mean pressure coefficients. The large values of positive mean pressures occur on the windward region; on the neighborhood surface of the top roof, negative pressures are found because of flow separation. On the leeward region, only small values of positive pressures exist, which are mainly caused by suction pressures on the opposite inner side of the roof. The contours of RMS pressure coefficients are shown in Fig. 3(b). RMS pressure coefficients have a uniform distribution compared with mean pressure coefficients. Relatively large RMS coefficients are found at the edge of the windward roof and the top roof as a result of flow separation. The values of RSM pressure coefficients are generally lower than those of mean wind pressure coefficients.
3 Application of Method of Equivalent Static Wind Loads

3.1 Mode Analysis of Structure

The first natural frequency is 1.39 Hz and about 30 mode shapes range from 1.39 to 5.51 Hz. Hence the natural frequencies are fairly close to one another, which is one of the most important considerations when analyzing the dynamic response of large-span roofs.

3.2 Computing Parameters and Computation of Peak Responses

Wind and structural parameters for computation are as follows: (1) terrain category: B; (2) 10 min averaged wind speed at a 10 m height: 35.8 m/s; and (3) structural damping ratio: 0.01. Time-domain analysis of dynamic responses was carried out using the fluctuating wind pressure obtained from the wind tunnel test. Based on the time series data gained from the time-domain analysis, mean and RMS responses were easily achieved. Peak responses were then obtained using Eq.(2), which can be used for the computation of equivalent static wind loads.

3.3 Characteristics of Equivalent Static Wind Loads

As mentioned, the case study only focuses on the 90° wind direction. For simplicity, 49 points on the upper surface of the roof were chosen for the analysis below. These points are uniformly distributed. For the structure, span-wise and vertical displacements play the most important roles in the structural design, corresponding to Y and Z displacements in the reference frame, respectively. Responses are selected from the same type of displacement to construct the group, in which the equivalent static loading is convenient for structural designers. Before grouping the responses, equivalent static loadings aiming only at a specific peak response are computed through the modified LRC method. Then, the similarity algorithm in Section 1.2 is applied and $\alpha$ is set as 0.5. Results of the grouping for displacement response are shown in Fig. 4. Generally, neighborhood points have a high possibility of forming the same group.

After grouping is completed, equivalent static wind loads for different groups (Fig. 5) can be computed using the method described in Section 1.3. For the symmetry characteristics of wind pressure and structure, only equivalent static wind loads for displacement group I–III are shown in Fig. 5. The symbol $P_{y,1}$ denotes the equivalent static wind distribution aiming at Y displacement responses in group I (the same as others). Equivalent static wind loads have somewhat similar characteristics of background component given by the traditional LRC method. The large pressure usually occurs in the region where the targeted group is located, partly caused by the large influence coefficients in the neighborhood of such a region. Another reason is the strong correlation between the responses of group and wind pressure in these regions. In contrast, pressures in the rest of the region are generally small. Nevertheless, not all equivalent static wind distributions in Fig. 5 have the obvious characteristics mentioned above. In Figs. 5, some large pressure levels could be seen on the windward surface of the roof.

Equivalent static wind loads obtained in the paper can, to some extent, provide an effective wind distribution similar to what the LRC method does. Fig. 5 shows that equivalent static wind loads are different between groups, indicating that equivalent static wind loads can only aim at peak responses within a certain group. In addition, the values of equivalent static wind loads fall in the range of -1.2~+2.8 KPa, indicating that the range of magnitude of equivalent static wind loads achieved in this study is analogous to that of natural wind pressure.

3.4 Comparison of Peak Responses

Peak responses [Eq. (2)] based on the time-domain analysis are regarded as precise results. To verify computational accuracy, peak displacements reproduced when the structure is subjected to equivalent static wind loads for groups are compared with the precise ones. Pictures
of correlations between the two results are plotted in Fig. 6, which correspond to Fig. 5. Each spot in the picture of correlations represents a certain response. The closer a spot is to the diagonal, the smaller the error between the reproduced and precise values. All reproduced responses in the group agree well with the precise values. This can be verified by certain spots located on the diagonal, which stand for the responses in the targeted group. If measurement indexes are calculated separately for the targeted responses above, we get $\theta=0^\circ$, $\lambda=1$ and $e=0.00\%$, indicating that the errors are zeros. However, when accuracy for the responses of non-targeted groups is evaluated, the result is not satisfactory. For example, when the structure is subjected to $P_{\alpha,1}$, the ratio of length $\lambda$ is 0.966 for Z displacements in Fig. 6(b), indicating that smaller errors may exist. Even in this situation [Fig. 6(b)], errors between the reproduced responses and precise ones reach around 20% ($e=25.60\%$). The method of equivalent static wind loads can indeed reproduce multi-responses in a certain group, which expands the application of equivalent static wind loads; however, this method cannot be used for determining all responses.

4 Concluding Remarks

As a new attempt to explore the calculation of ESWLs for large-span roof structures, the grouping response method is proposed herein to reproduce grouped peak responses simultaneously. In this method, similarity algorithm is used to group the responses for an actual large-span roof. Then, equivalent static wind loads are calculated based on a modified LRC method. When the similarity of responses in the same group is high, equivalent static wind loads with high accuracy and reasonable magnitude can be achieved. A balance among the number of targeted responses, computational accuracy, and magnitude of equivalent static wind loads is constantly observed. Indeed, the method could bring some errors for non-targeted groups though the results in targeted group are satisfactory.

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References


Fig. 4 Response grouping for displacement

(a) Group for Y displacement

(b) Group for Z displacement

(a) $P_{y_1}$

(b) $P_{z_1}$

(c) $P_{y_{II}}$

(d) $P_{z_{II}}$
Fig. 5 Equivalent static wind loads for different displacement groups (unit: KPa)

\(\theta=53.405^\circ, \lambda=0.5507, e=80.42\%\)

\(\theta=14.834^\circ, \lambda=0.9660, e=25.60\%\)

\(\theta=39.417^\circ, \lambda=0.5942, e=65.95\%\)

\(\theta=21.530^\circ, \lambda=0.6814, e=44.34\%\)
Fig. 6 Correlations between displacement peak responses by time-domain analysis and ones reproduced by ESWL for different groups.