Numerical Simulation of Unsteady 3-D Flow around
a Yawed and Inclined Circular Cylinder

DongHun Yeo\textsuperscript{a} & Nicholas P. Jones \textsuperscript{b}

\textsuperscript{a}University of Illinois at Urbana-Champaign, 205 N. Mathews Ave. Urbana, IL, USA
\textsuperscript{b}Johns Hopkins University, 3400 N. Charles St. Baltimore, MD, USA

ABSTRACT: A three-dimensional Detached Eddy Simulation (DES) study was conducted to investigate the three-dimensional characteristics of the fully-developed flow past a yawed and inclined circular cylinder. We simulated the flow at Reynolds number (Re) of $1.4 \times 10^5$ (based on freestream velocity $U$ and the diameter of the cylinder $D$) over a cylinder with an axial length of 20D. The simulation shows that swirling flow structures with low pressure are developed along the cylinder axis. As a result, a fluctuating force is generated on the cylinder surface with frequency much lower than that of Karman vortex shedding.

KEYWORDS: yawed and inclined circular cylinder, DES, three-dimensional flow.

1 INTRODUCTION

Flow oblique to a bluff body has complex three-dimensional characteristics due to both the flow structures and the geometry of the body. The flow past a yawed and inclined circular cylinder has been of significant interest to the engineering community. One of the applications is large-amplitude and low-frequency vibration of the stays of cable-stayed bridges induced by oblique wind. Much research on the excitation mechanism has been conducted, but complete understanding of the underlying mechanism remains elusive.

Theoretical and experimental studies in early research attempted to show that the flow around a yawed cylinder can be associated with that of a non-yawed cylinder by the component of the freestream flow normal to the cylinder axis [1]. This is known as the Independence Principle or the Cosine Rule. Several experimental investigations assessed deviations from the prediction of the Independence Principle [2]. However, experiments have inherent difficulties in identifying and visualizing the three-dimensional complex flow structures around an oblique cylinder.

Numerical studies of the phenomenon have been restricted to low Reynolds number flows due to the availability of computational resources and appropriate numerical methods. Kawamura and Hayashi [3] simulated the flow around a yawed circular cylinder at $Re = 2,000$ based on a finite difference method (FDM) and observed a significant effect of spanwise boundary conditions. Lucor and Karniadakis [4] applied direct numerical simulation (DNS) to study of flow past a yawed circular cylinder at $Re = 1,000$ and presented a flow structure that featured traveling inclined "braids"; detailed explanations, however, were not provided.

To overcome the limitations from both experiments and numerical simulations, we apply the three-dimensional DES approach to a simulation of the flow. DES enables simulation of unsteady three-dimensional flow at high Reynolds number while maintaining reasonable computational requirements to improve the understanding of the complex characteristics of the flow.

While there have been many efforts to investigate the characteristics of cylinder flow, this is, to the authors' knowledge, the first study to focus on the three-dimensional characteristics of the fully-developed flow at high Reynolds number around a yawed and inclined circular cylinder using numerical simulation.
2 NUMERICAL SIMULATION

2.1 Spalart-Allmaras DES approach

Spalart et al. [5] developed the concept of DES, which is a hybrid method of combining the strengths of Reynolds-Averaged Navier Stokes equations (RANS) in the boundary layer region and of Large Eddy Simulation (LES) in the highly separated flow region by modifying the turbulence length scale in the Spalart-Allmaras (S-A) RANS turbulence model. The transition between RANS and LES is controlled by the grid spacing and the wall distance. The RANS region in DES alleviates the near-wall resolution requirement, and therefore DES can simulate highly separated flows at high Reynolds numbers at a manageable computational cost.

2.2 Definition of angles

The definitions of yaw and inclination angle of a cylinder are shown in Figure 1. The yaw angle $\beta$ is defined herein as the angle between a horizontally skewed cylinder and an axis oriented normal to the incoming flow, and the inclination angle $\theta$ is defined as the angle between a vertically sloped cylinder and its horizontal axis.

![Figure 1. A yawed and inclined cylinder](image)

2.3 Numerical Methods

Three-dimensional DES with the S-A DES model was conducted for a flow at $Re=140,000$ past a yawed and inclined circular cylinder using the WIND-US CFD code [6]. The code uses a cell-vortexed finite-volume approach and has been being developed by the NPARC (National Project for Application-oriented Research in CFD) Alliance.

The computational domain had an extent of $40D$ in the streamwise direction ($20D$ each in the upstream and downstream region), $40D$ in the cross-stream direction ($20D$ from the center of the cylinder to the top and bottom side), and $20D$ cylinder length in the spanwise direction.

The simulation employed a constant velocity for the upstream boundary, zeroth order extrapolation with fixed static pressure for the downstream boundary, inviscid condition for the top and bottom boundary, and periodic condition for the spanwise wall boundary to avoid end effects, which potentially significantly affect three-dimensional flow structures. Since the first grid normal to the cylinder surface was located around wall unit $y^+$ (this non-dimensional viscous length scale is based on the viscosity and wall shear stress of a flow) of 1, a no-slip condition on the surface was employed without wall treatments. A spanwise grid $\Delta Z$ in the global cross-stream dimension was $0.1D$. 129 grids were employed on the cylinder surface along the perimeter; The total size of the grids was about $3.0 \times 10^6$ elements. The numerical scheme used a $5^{th}$ order upwind scheme for convection, a $2^{nd}$ order central scheme for viscosity, and a two-step MacCormack implicit scheme for time integration with 4 Newton sub-iterations per timestep. A non-dimensional timestep of 0.01 (calculated as $\Delta t^* = Ut/D$) was used.

The simulation assumed turbulent separation from the cylinder surface, and the analysis did not consider the issue of boundary layer transition: the $Re$ of 140,000 is in the sub-critical range.
3 RESULTS

A 30-degree yawed and 45-degree inclined circular cylinder was simulated. Figure 2 shows two cases at different times using the iso-surface of the second invariant Q of the velocity gradient tensor [7], which identifies coherent vortex structures. The flow direction is from left to right. Figure 2 indicates that a flow structure with intense vorticity (as indicated by arrow) delays the detachment of vortex structures and moves along the cylinder axis.

Figure 3 is the result of a fast Fourier transform (FFT) analysis of the force coefficient Cy (based on force in the local y axis of cells along the cylinder perimeter and free stream velocity) at non-dimensional cylinder length z/D=10. Compared to the reduced frequency (based on free stream velocity and the cylinder diameter) of the Karman vortex shedding (fr2 \( \approx 0.197 \)), the fluctuation frequency of a flow structure is 6 times lower (fr1 \( \approx 0.033 \)). The velocity of the low-frequency flow structure is about 86% of the velocity component of the upstream flow in the cylinder axis direction.

Figure 4 confirms the existence of two almost identical flow structures using correlation coefficients calculated from the time-dependent force coefficients from two positions along the cylinder axis. They are always in phase and move along the cylinder axis at the same velocity.

Figure 5 shows the spatial distribution of the force coefficients of the cylinder in the local coordinates versus time. It clearly suggests that two three-dimensional flow structures are developed and that local forces move along the cylinder at a constant velocity.

Figure 6 illustrates how the flow structures with low frequency occur. The flow direction is out of the page, and the view is from behind the cylinder. Incoming flow beyond the separation line of the cylinder surface develops rolled-up shear layer flow with a large spanwise velocity co-
mponent. These swirling eddies grow and become a coherent flow structure of finite length with vorticity and a velocity component in the cylinder axis. These swirling flows are developed from the top and the bottom of the cylinder in turn along the cylinder and move to the center line obliquely. They generate in order such that the structure of “1” develops from the bottom of the cylinder, moves obliquely to the centerline and dissipates, and that of “2” comes from the top, goes to the centerline obliquely and disappears, and so on. The path of the swirling flow structures therefore has a “zigzagged” pattern along the cylinder axis. During that time, they pull in the Karman vortex structures near the cylinder, delay the detachment and therefore develop high vorticity. This induces moderately large forces on the local surface of the cylinder. These sequential flow structures create moving forces along the cylinder axis at a specific velocity, with the frequency of fluctuating forces being much lower than that of Karman vortex shedding.

4 SUMMARY

DES was applied to the fully-developed three-dimensional flow at high Reynolds number past a yawed and inclined circular cylinder. The results indicated swirling eddies grow from rolled-up shear layer flow and develop a coherent flow structure with vorticity and an axial velocity component. This swirling flow structure with finite length moves obliquely to the centerline from the top and bottom of the cylinder in turn. While they move alternatively along the cylinder at a specific velocity, the low pressure of the flow structures behind the cylinder delays the detachment of the Karman vortex structures and makes them more intense. Therefore the cylinder is subject to fluctuating forces with low frequency induced by the movement of these structures along the cylinder. This mechanism demonstrates three-dimensional flow past a yawed and inclined circular cylinder, and provides insight into understanding the characteristics of the flow.

5 REFERENCES