A Numerical Study of Aerodynamic Forces on a Square Cylinder in Oscillating Flows

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ABSTRACT: The unsteady forces on a square cylinder in sinuso idally oscillating flows with nonzero mean velocities are investigated numerically in a two-dimensional sense. The major parameters in the analysis of flows around a cylinder with a sinusoidal motion are the Keulegan-Carpenter number ($KC$) and the ratio ($AR$) between the variation and the mean of the approaching-flow velocities, which vary respectively from 1 to 30 and 0.1 to 0.7. The resulting unsteady forces on the cylinder for various $KC$ and $AR$ values are analyzed systematically to examine the flow effect of the cylinder. Based on the numerical results, additionally, the Morison equation is used as a basis to describe the variations of resulting drag and lift.

KEYWORDS: Numerical simulation, Square cylinder, Morrison equation

1 INTRODUCTION
Due to the unsteady essence of natural wind, the problem of unsteady fluid forces on a bluff body in oscillating flows becomes an interesting subject in wind engineering. Considering the fluid dynamics of bodies in an oscillatory flow, there are fundamental differences of vortex motions and induced forces between the cases of a steady uniform flow and an oscillatory flow. There are many aspects of fluid forces induced by oscillating flow and need extensive investigations since there have been many examples of structure damage caused by such forces.

2 THEORETICAL BACKGROUND
For a sinusoidal flow past a cylinder, Morison \textit{et al.} proposed a set of theoretical descriptions about the variations drag and lift forces on the body as follows: \[1\]
\[
F_D = \frac{1}{2} \rho D \bar{C}_D U |U| + \rho A \bar{C}_D \frac{dU}{dt}
\]
\[
F_L = \frac{1}{2} \rho D \bar{C}_L U |U| + \rho A \bar{C}_L \frac{dU}{dt}
\]
with $U(t) = U_0 + U_m \sin(2\pi t / T)$ \[2\]
where $U_0$ and $U_m$ are the mean and amplitude speeds of the approaching flow; $T$ is the oscillation period; $A$ and $D$ are the cross-sectional area and width of the cylinder; $\bar{C}_D$ and $\bar{C}_L$ denote the mean drag and lift coefficients; $\bar{C}_D$ and $\bar{C}_L$ are the inertia coefficients of the unsteady drag and lift forces. It is noted that Eq. (2) is valid only when the approaching-flow attack angle is not zero.

In the study, an important parameter to describe the variation of the oscillatory approaching flow is the Keulegan-Carpenter number \[2\], defined as $KC=U_m T / D$. It has been widely used to describe the variation pattern of the unsteady approaching flow.
3 PROBLEM DESCRIPTION

As shown in Figure 1, the problem of a sinusoidal flow past a square cylinder at a high Reynold's number ($Re = U_0 D/\nu = 10^7$; $\nu$ is the fluid viscosity) is investigated. By varying the $KC$ value (1 to 30) and the ratio of the velocity variation ($AR=U_m/U_0$) of the approaching flow (0.1 to 0.7) at three attack angles ($\theta = 0^\circ$, 22.5$^\circ$ and 45$^\circ$), the resulting unsteady forces on the cylinder are analyzed to gain additional insight into the analysis of related engineering problems.

4 NUMERICAL METHOD

The simulations adopt the weakly-compressible-flow method [3] together with a dynamic subgrid-scale turbulence model [4]. The flow results are solved implicitly according to a finite-volume approach and a Crank-Nicolson scheme [5]. In all the computations, the domain of flow calculations, as depicted in Fig. 1, is in a rectangular shape (22D×13D with a grid size of 266×96).

5 RESULTS

Based on the calculated time series of the drag and lift, the mean and inertia force coefficients are determined by performing least-square regressions according to Eqs. (1) and (2). Figure 2 shows the predicted force coefficients compared to those from the experiments by Okajima et al. [6] and Bearman et al. [7]. It can be seen that good agreements are obtained.

Although the complete program includes three attack angles, the results of only two selected cases ($\theta = 0^\circ$ and 45$^\circ$) are presented. At a zero attack angle, Fig. 3 shows the variations of the predicted mean and inertia drag coefficients. Except for relatively mild variations as $AR=0.1$, $CD$ generally decreases at the beginning then increases to a peak value as $KC$ is about 10 (Fig. 3a). After that, it decreases again then reaches a second but milder peak as $KC$ is about 20. In terms of $\tilde{C}_D$ (Fig. 3b), on the other hand, two peak values are also obtained at the same two $KC$ values (10 and 20). In most cases (except when $AR=0.1$), a larger $AR$ results in greater $CD$ and $\tilde{C}_D$ values.

When $\theta = 45^\circ$, the tendencies of the variations of $\tilde{C}_D$ and $\tilde{C}_D$ at different $KC$'s are similar to those in the case $\theta = 0^\circ$ (Fig. 4). Figure 5 shows additionally the resulting mean and inertia lift coefficients. Compared to those of the drag coefficients, the deviations of the lift coefficients appear relatively smaller. Two peak values are again detected as $KC$ is about 10 and 20.

6 DISCUSSION

The agreement between the experimental and numerical results (Fig. 2) shows that the numerical method can be applied to predict the flow with good accuracy. Accordingly, one can then use the results in Figs. 3 to 5 to evaluate the forces on the cylinder for design purposes.

The occurrence of peaks is apparently due to the effect of resonance between the oscillation of the approaching-flow and the motion of vortex shedding of the cylinder. To clarify the detailed mechanism, extensive analyses have been made in detail. Due to the limitation of the paper space, however, further discussions and comments are not able to be made presently.

7 CONCLUDING REMARKS

The unsteady forces on a square cylinder at three selected attack angles in sinusoidal approaching flows with non-zero mean velocities are investigated systematically based on the numerical results. The effect of the approaching-flow unsteadiness, which is determined by the $KC$ and $AR$ values, is discussed according to the theoretical relationships of Morison's equations. Finally, related forces coefficients are presented in help with the analysis of engineering designs.
8 REFERENCES

Figure 1. Sketch of the problem.

Figure 2. Comparison of force coefficients at various KC values.
Figure 3. Mean and inertia drag coefficients at various KC values. ($\theta = 0^\circ$)

Figure 4. Mean and inertia drag coefficients at various KC values. ($\theta = 45^\circ$)

Figure 5. Mean and inertia lift coefficients at various KC values. ($\theta = 45^\circ$)