Numerical Calculation of the 3-Dimensional Motion of Wind-borne Debris

Peter J Richards\textsuperscript{a}, Nathan Williams\textsuperscript{a}, Brent Laing\textsuperscript{a}, Matthew McCarty\textsuperscript{a}, Michael Pond\textsuperscript{a}

\textsuperscript{a} Department of Mechanical Engineering University of Auckland, Private Bag 92019, Auckland, New Zealand

ABSTRACT: A 6-degree-of-freedom trajectory model is developed which makes use of complex aerodynamic data obtained from wind tunnel testing of both plate and rod type debris. It is shown that the complex computed trajectories are similar to those observed in model tests.

KEYWORDS: Trajectory, Debris.

1 INTRODUCTION

During extreme storms the damage of a particular building may be strongly influenced by the impact of debris that has been generated by the failure of other structures upstream. It is hence important to consider what types of debris are likely to occur, what are their likely masses, how far they are likely to travel and what fraction of the wind speed they are likely to attain before impact. Wind-borne debris has often been classified into compact, rod and plate types, where compact debris has three similar dimensions, rod type has one dimension much larger than the other two and plate type has one dimension much smaller than the other two. For compact debris the trajectories may be considered to be planar, but with both rod type and plate type debris the real trajectories are three-dimensional and include significant tumbling actions. Hence trajectory calculations may need to include the full 6-degree-of-freedom motion.

Most previous studies of plate type wind-borne debris have only considered 2-dimensional motion. While such trajectories can demonstrate many basic principles, planar motion is a special case that is unlikely to occur in a real storm situation. In these cases the normal force coefficient and the position of the centre of pressure are only functions of the angle of attack. In addition the centre of pressure is always located on the line of symmetry. However the real situation is significantly more complex.

Wind tunnel measurements at the University of Auckland, for both plate and rod type debris, have sought to provide force and moment information for more general situations including all possible orientations of several generic shapes. This aerodynamic data has then been incorporated into a 6-degree-of-freedom (6 DoF) trajectory model.

2 EQUATIONS OF MOTION

Numerical solutions have been obtained for the 6-DoF motion of a general rectangular object. The orientation of the principal axes are chosen such that \( l_x \leq l_y \leq l_z \), as illustrated in Figure 1(a).
The object is considered to be travelling with respect to the earth fixed axes $X_e$, $Y_e$, $Z_e$ with its centre of gravity $G$ at position $X=[x_e, y_e, z_e]$ and moving with velocity $V=[V_X, V_Y, V_Z]$, as illustrated in Figure 1(b). $X$ and $V$ are related through the obvious equation

$$V = \frac{dX}{dt}$$

Allowance is made for a wind of velocity $W = [W_X, W_Y, W_Z]$, where in general $W=W(x_e, y_e, z_e, t)$, that is the wind is a function of position and time. In this paper the wind will be assumed to be steady and uniform but the solution routine has made provision for any predefined wind field, so that in future developments the effects of turbulence, the velocity gradient or the wake of a building may be considered. The velocity of the object relative to the air is therefore

$$U_G = V - W$$

The translating axes $X_G$, $Y_G$, $Z_G$ are parallel to the earth fixed axes but move with the object. Further the position of the principal axes of the body $X_P$, $Y_P$, $Z_P$ are defined by the angles $\beta$, $\alpha$ and $\phi$ as illustrated in Figure 1(c). In order to calculate the forces on the body the relative velocity is transformed into principal axis components. Then once the body oriented aerodynamic forces, $F_p=[F_{XP}, F_{YP}, F_{ZP}]$, are determined, these are transformed back and used in the acceleration equation

$$m \frac{dV}{dt} = F_G - mgj.$$  

The rotation calculations are most easily handled in terms of the angular momentum vector referenced to the body’s principal axes $\mathbf{H}_p=[I_{XX}p, I_{YY}q, I_{ZZ}r]$. Where the angular velocity vector

$$\mathbf{\Omega} = [p, q, r] = [\dot{\phi} + \beta \sin(\alpha), \dot{\alpha} \sin(\phi) + \dot{\beta} \cos(\alpha) \cos(\phi), \dot{\alpha} \cos(\phi) - \dot{\beta} \cos(\alpha) \sin(\phi)].$$  

Euler’s equation may then be written as

$$\frac{d\mathbf{H}_p}{dt} = \mathbf{M}_p - \mathbf{\Omega} \times \mathbf{H}_p$$

Equation (5) is integrated to provide the changes in angular velocity, whereupon Equation (4) is rearranged to give the rates of change of the angles. Equations (1, 3, 4 and 5) are numerically integrated using the Improved Euler’s Method in order to simulate 6-degree-of-freedom motion.
3 FORCES AND MOMENT

Wind tunnel measurements of the forces and moments acting on plates with side length ratios \( l_z/l_y = 1, 2 \) and 4 and on long rods with cross-section side ratios \( l_y/l_x = 1, 2 \) and 3 have been conducted in the University of Auckland Twisted Flow Wind Tunnel. Figure 2 shows an example of the normal forces and centre of pressure positions for a plate with \( l_z/l_y = 2 \). The normal force coefficient of 1.2 for situations where the flow is nearly perpendicular to the plate is close to that given by other researchers. However the sensitivity to tilt angle \( \gamma \) when the angle of attack \( \varepsilon \) is about \( 30^\circ \) is less well known. The normal force coefficients have been incorporated into the analysis by using bi-linear interpolation. Analysis of the moments showed that the centre of pressure moved from approximately the quarter chord point at low angles of attack towards the centre at high angles. However it was also found that the position was displaced towards the major axis on the 4:1 and 2:1 plates. For modelling purposes the centre of pressure is located at

\[
[y_{CoP}, z_{CoP}] = \left[ \frac{c}{4} \left( \frac{\pi}{2} - |\varepsilon| \right) \sin(\xi), \frac{c}{4} \left( \frac{\pi}{2} - |\varepsilon| \right) \cos(\xi) \right]
\]

where \( \tan(\xi) = \frac{\tan(\gamma)}{l_z/l_y} \) and the chord length \( c = \frac{l_z \times l_y}{l_z \cos(\gamma) + l_y \sin(\gamma)} \).

For plate type debris the apparent camber caused by rotation is accounted for by using

\[
\Delta C_N = \frac{2\pi}{1 + 2/AR} \min \left( \frac{d\varepsilon}{dt} c \cos(\varepsilon), \left( \frac{2h}{c} \right)_{\text{max}} \right)
\]

where \( AR \) is the aspect ratio of the plate at the particular orientation. Since this term can’t increase indefinitely an upper limit \( (2h/c)_{\text{max}} \) is specified.

With plate type objects only the normal \((X_P\) direction) force is significant, but with rod type objects the \( Y_P \) direction force is also important. Figure 3(a) shows the \( Y_P \) direction force coefficients for a square section rod. In this case \( C_Y \) is zero if either the tilt \((\gamma)\) or yaw \((90^\circ-\varepsilon)\) angles are zero since the flow will be symmetric about the \( X_PZ_P \) plane. This force coefficient is a maximum when both the tilt and yaw angles are \( 90^\circ \) since the flow is then parallel to the \( Y_P \) axis. It is interesting to note that at small yaw angles the \( Y_P \) direction force is negative, this was found to be a nuisance during wind tunnel testing since at times it caused a galloping type vibration.

![Figure 2(a) Normal force coefficients for a plate with side length ratio 2 and (b) the centre of pressure locations.](image)
Figure 3(a) Yf direction force coefficients for a square rod, (b) the wind tunnel arrangement for the free flight tests.

4 SAMPLE TRAJECTORY

Figure 3(b) shows the wind tunnel arrangement used to record free flight tests of model scale debris. As an example of the 6-DoF motion Figure 4 shows a 1:10 scale model of a roofing sheet with an area of 2.25 m², side length ratio of 2, mass per unit area of 3.4 kg/m² and a wind speed of 31.6 m/s. The 1/10th scale model has an area of 0.0225 m², a mass per unit area of 0.34 kg/m² and the wind speed was 10 m/s. The particular test case was released with $\alpha = 30^\circ$, $\beta = 20^\circ$ and $\phi = 30^\circ$. Figures 4(a) and (c) show composite video images, where unfortunately the side view images were very blurred, while Figures 4(b) and (d) show the corresponding calculated trajectory. While there isn’t an exact match the numerical model shows a similar pattern of rotations.

5 CONCLUSIONS

A 6-degree-of-freedom trajectory model has been created which makes use of complex aerodynamic data. It has been shown that the computed trajectories, which include movements in all three directions and rotations about all three axes, are similar to those observed in model tests.

Figure 4. Top and side views of the motion of a 2:1 side ratio plate as captured on video and computed.