Computational Methods for the Wind Energy Industry

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ABSTRACT: A brief history of the development of models used in wind resource calculations and how they are used within that role is given. The earliest models were based on linear theory and were valid only over a narrow steepness and roughness parameter space. Developments in the intervening period have seen this parameter space expanded by using nonlinear Reynolds averaged models with distributed drag parameterizations for modelling mean flow within and above canopies. Though mean flow results for moderate slopes can be accurate, the effects of hill-scale turbulence created and shed on the lee side of hills becomes important as slopes increase. Large Eddy Simulation (LES) and wind tunnel data are used to help understand the processes responsible for the creation and downwind propagation of eddies in the lee of steep topographic features with and without canopies.

KEYWORDS: wind resource modelling turbulence separation LES

1 INTRODUCTION

Computational methods within the wind industry have a broad range of uses but generally fall into two areas - aeroelastic aspects of wind turbine design and wind resource assessment. The former activity is embedded primarily within the engineering domain with the latter having its development and natural home in meteorology. In many senses there is a separation between these two that results naturally from the length and time scales considered. However this separation is becoming less clear as the use of meteorological modelling moves its focus down-scale, with eddy resolving calculations producing turbulent fluctuations with length scales of less than the length of currently manufactured turbine blades. To some extent this progress toward higher resolution computations is being driven by the need to determine levels of turbulence at turbine locations. This is in addition to the standard determination of the mean wind distributions using Reynolds averaged models. These are in turn clearly driven by commercial imperatives affecting revenue generation and the cost of the wind farm operations and maintenance. There are also other differences between these two areas of research and development that have arisen due to their needs to deal with geometrically and topologically different applications. For example, meteorological models have traditionally used regular grids and have typically used finite-difference or finite-volume formulations. This is in contrast to more typical irregular grids associated with finite-element codes for complex geometries found in turbine blade design. But again these differences are diminishing as more effort is being put toward development of meteorological models aimed at yielding higher and higher resolution calculations in more complex geometries.

This paper focuses on the fine-scale meteorological end of the research and development spectrum with a forward-looking view to how tools and models currently at the leading edge of technology development are likely to find their way into mainstream wind energy calculations and even turbine design. Though far from comprehensive, this paper gives a historical and
theoretical context for the development of models currently in common use for wind resource assessment and begins to set the stage for turbulence prediction in the context of wind turbine and wind farm design.

2 THE ROLE OF CFD IN MODELLING WIND RESOURCE

Few would argue with any conviction that the atmosphere is a simple physical system. Measurements in the atmosphere contain information at time scales ranging from inter-annual to fractions of seconds and spatial scales from global to sub-millimetre. Though history has shown predictions of future computing power to be fraught with inaccuracy, it nonetheless seems safe to forecast that for the foreseeable future we will not have a model capable of simultaneously resolving all of these length and time scales. Fortunately for the wind industry, and the field of industrial aerodynamics in general, there are many regions of the broad atmospheric parameter space that yield flow solutions in which certain processes and their concomitant length and time scales dominate.

One example of this is the European Wind Atlas methodology (see Troen and Petersen, 1989). This is essentially a method of vertical and horizontal extrapolation away from a measurement taken somewhere within or near a proposed wind farm, using steady state flow solutions. This methodology is in common use within the wind industry. The process begins by dividing the wind measurements into several (typically 12 or more) sectors. A steady state calculation is then done for wind directions corresponding to each of these directions/sectors over a model domain containing the measurement location. Based on the flow solutions, topographic and roughness perturbations are calculated for each of the sectors at the measurement and prediction locations (see Figure 1). For linear models, the background solution from which the perturbations are calculated is explicitly specified with the model producing perturbations from that background (upstream) flow, while for nonlinear calculations the process involves calculation of a layer average from which perturbations are then calculated. By linearly scaling the speed of the flow at the measurement location, a horizontally homogeneous background flow representative of an unperturbed value in the absence of any topography or roughness change is determined (A in Figure 1). After extrapolating vertically from the measurement height to the prediction height (if they are different, B in Figure 1), the perturbation at the prediction location is then used to move from the calculated background value to the predicted value at the prediction height and location (C in Figure 1).

![Figure 1 Schematic view of the European Wind Atlas methodology.](image-url)
A number of assumptions are used in this methodology. The first is that the model used adequately represents a sufficiently broad parameter space to recreate the perturbations in the flow caused by roughness and topography. This is not always the case as most models used in this context are relatively simple, many linear in their formulation. The second is that the background flow does not change in the mean sense over the distance between the measurement point and the prediction point. This can clearly be in error where mesoscale processes not represented in the model used, are present. It also assumes that there is coherence in the flow between the measurement and prediction points. This is equivalent to assuming a correlation coefficient of 1.0 between measurements that would be taken at those two points. In an effort to identify and quantify some of the above, Ayotte et al. (2001) used band limited spectral correlation analysis to show that for wind resource assessment, over small distances the last two of these assumptions hold up reasonably well. Using data from a number of tall meteorological masts separated by varying distances, the work also quantified the degradation of accuracy with distance over which predictions were made. This work showed that, in a practical sense, typically there is an upper limit of a few kilometers on the distance over which measurements can be extrapolated away from the measurement location.

3 MODELS USED IN THE EUROPEAN WIND ATLAS METHODOLOGY

3.1 Linear flow models

Wood (2000) noted that the theory of flow over hills had its origins over fifty years ago. In his review, a comprehensive history of the development of the theory of turbulent flow over hills was given. Much of the theory of flow over small hills, those often encountered in day-to-day wind resource modelling is linear, and originates with the theoretical treatments of Jackson and Hunt (1975) and Hunt et al. (1988). As such it was natural for linearised flow models for commercial use to emerge from the theoretical and computational work on turbulent flow over hills. In these models the equations of motion were simplified by linearising the advection terms and the other weaker nonlinearities in the turbulence closure equations. The linearisation of the advection terms was done by assuming the background flow was constant in the horizontal or terrain-following directions but varied vertically. This vertical variation often consisted of three or more layers corresponding to a division of the turbulent boundary layer into separate layers where different balances of terms in the governing equations were dominant. After Hunt et al. (1988), Figure 2 shows the flow over an isolated hill divided into four layers. The lowest layer, the Inner Surface Layer (IS), is a thin layer next to the surface and contains the roughness sublayer where velocity goes to zero at the surface and viscous forces dominate. Above this is the Shear Stress (SS) layer where the flow is strongly sheared and turbulent stress divergence strongly affects the mean flow. It is within this layer that the maximum speedup in the wind profile occurs. The Outer Region is made up of two more layers, the Middle (M) and Upper (U) layers in which inertial forces dominate. The Middle layer is inviscid and rotational with the Upper (U) layer inviscid and irrotational. Examples of models based on this layered theory were the WAsP model (see Troen and Petersen, 1989) or MS-Micro (see Taylor et al., 1983). In these models the various layers were asymptotically matched in the vertical to yield a full solution for the accelerations and decelerations of flow over low hills. As these layers were to some degree defined by the mathematical methods that were applied to them, the resulting vertical structure of the flow was limited in the form it could take and as such did not always
match well with the vertical structure of a more realistic atmosphere. Neutral stratification was also assumed in these models, though some progress was made in introducing a limited treatment of stratification to this type of model.

Figure 2 Layered flow over a hill after Hunt et al. (1988).

The earliest widely used layered linear models were formulated in Fourier-Bessel space (see Troen and Petersen, 1989) or two dimensional Fourier space (see Salmon and Taylor, 1983) in the two terrain following coordinates. At least in part this was due to the analytical methods used to solve the flow equations within the model. In one of a number of developments aimed at broadening the parameter space within which these linear models gave accurate solutions, Beljaars et al. (1987) formulated a linear mixed spectral-finite difference (MSFD) model in which the terrain-following dimensions were spectral (Fourier modes) and the multi-layered analytic formulation was replaced by a finite difference, boundary value solution of the equations for the mean and turbulent flow. Rather than imposing a layered structure on the solution in which various analytical solutions could be used, this type of model accomplished a natural smooth transition between these regions in the flow. Over a number of years the MSFD model was modified to include different vertical numerics (Karpic, 1988) and more complete turbulence closure physics (Ayotte et al., 1994, Ayotte and Taylor, 1995). In addition, limited progress was made by Ayotte (1997) in using variational optimisation to create upstream unperturbed wind profiles that were optimal fits to upstream wind profile data, while being constrained by the model equations. This was shown to increase the accuracy of such calculations under a limited range of circumstances.

Though the mathematical underpinning of these models was complex, their implementation was computationally inexpensive and fairly robust. When used within the parameter space for which they were designed – specifically neutral flow over low, smooth hills where the background flow could be well approximated within the model, they worked well and served the wind industry for many years. In fact they are still in use today. In comparisons with wind tunnel measurements from flows carefully created to stay within that limited parameter space, the models preformed well (Salmon et al., 1988). Similarly, when atmospheric measurements such as those from Askervein Hill (Taylor and Teunissen, 1987) were filtered to include only neutrally stratified, well mixed flows, linear models performed reasonably well with noted over-estimates of the hill-top acceleration and under-estimates of the lee-side decelerations. Both of these failures to reproduce measurements can be attributed primarily to the linear assumptions used in these models.

—98—
3.2 Nonlinear flow modelling

The above noted limited conditions under which these linear models can be expected to perform well represents a narrow slice of the steepness-roughness parameter space found in the real atmospheric boundary layer. Specifically, this parameter space is inclusive of only neutrally stratified flow over terrain of modest slope (< ~0.2), over relatively smooth surfaces (small values of $z_0$). These early models were simple by design and necessity. That necessity was brought about by the lack of computing power and, to a lesser degree, limitations in theory and numerical methods. Progress since then has been made on expanding that parameter space to include more complex situations typically found in wind farm design.

Though the wind industry still makes use of the linear models described above, because of the exponential growth of computing power and the development of commodity priced cluster and network technology, nonlinear models are beginning to see some use for wind resource calculations. These models are time dependent models that are integrated to a steady state in each of a number of direction sectors, typically 12 sectors covering the compass. These nonlinear solutions are then used within the standard European Wind Atlas methodology described above. Though not numerous, models used for this purpose exist at a number of research institutes and within a few commercial enterprises. An example of the former is the Blasius model developed by Wood and Mason (1993), created and used as a general purpose model for investigation of a broad range of atmospheric phenomena. An example of the latter is the Raptor$_{NL}$ model developed at Windlab Systems which will be described here recognising that there are likely as many unique formulations for models of this type as there are models.

As the Raptor$_{NL}$ model was designed specifically for use within the European Wind Atlas Methodology, it has a strong focus on rapid convergence to a steady state and accuracy over a limited parameter space in that it calculates only neutrally stratified turbulent flow over topography. The model is in some senses an extension of the linear model developed in Ayotte and Taylor (1995) in that it shares a number of the features associated with the turbulence closure and its implementation. For example it uses the same basis for the turbulence closure (Launder et al., 1975) and a similar method for the automation of the coding of the turbulence closure from Ayotte (1995). However, the model also differs significantly in that it is nonlinear, time-dependent and finite difference in all three coordinate directions rather than spectral in the two terrain following directions like that described in Ayotte and Taylor (1995). The model solves the incompressible, Reynolds averaged Navier Stokes equations,

$$\frac{\partial \bar{U}_i}{\partial t} + \bar{U}_j \frac{\partial \bar{U}_i}{\partial \bar{x}_j} = -\frac{1}{\rho} \frac{\partial p}{\partial \bar{x}_i} \frac{\partial \bar{u}_i \bar{u}_j}{\partial \bar{x}_j}$$  \hspace{1cm} (1)$$

$$\frac{\partial \bar{U}_i}{\partial \bar{x}_i} = 0$$  \hspace{1cm} (2)$$

where $U_j$ are the Reynolds averaged components of the mean flow, $p$ is pressure, $\rho$ is density and $u_i u_j$ with an overbar are turbulent stresses. This formulation includes both vertical and horizontal turbulent flux divergence. Equations for each of the six Reynolds stresses and dissipation are solved at each time step to close the mean equations. The model equations are solved using a fractional step method in which the equation for pressure is derived in such a way that it ensures a divergence-free momentum field.
Each of the Reynolds stress equations and the dissipation equation is solved separately as boundary value problems over each grid column. Off-diagonal terms such as advection and horizontal derivatives are calculated at the previous time step and treated as source terms in the tri-diagonal linear system. As in Ayotte and Taylor (1995) and Ayotte (1995), automated source code generation methods employing symbolic manipulation and parsing are employed to create the source code for the turbulence equations. However, tests have shown that if only the mean components of the flow are of interest, a simplified turbulent kinetic energy – dissipation closure gives very similar results to the full second-order closure. As such, for most commercial work, the latter is used.

The model is formulated in nonorthogonal terrain following coordinates similar to that of Wood and Mason (1993). The vertical coordinate is transformed into logarithmic space to ensure sufficient resolution within and immediately above the inner layer. Vertical velocity is transformed such that it is the rate of change of a particle’s vertical position with respect to the transformed vertical coordinate. The model uses a staggered finite difference grid and uses periodic boundary conditions in each of the terrain following coordinates.

The pressure equation and each of the momentum equations require the solution of a large, sparse linear system. These linear systems are inverted using a Krylov subspace projection method (GMRES) (Thomas, 1998, Saad, 2000). Solution of the pressure equation involves the use of a preconditioner that makes use of solutions (in Fourier space) of the elliptic pressure equation simplified by formulating it in orthogonal coordinates, ignoring the off-diagonal terms in the nonorthogonal coordinate transform.

### 3.3 Comparison to Measurements

Ayotte and Hughes (2004) made extensive wind tunnel measurements over isolated sinusoidal ridges of varying slope and roughness. The objective of this work was twofold. The first was to obtain a data set against which the RaptorNL model could be compared. The second was to examine the process of separation. Two surface types were used, one relatively smooth and the other relatively rough. The hill slopes were chosen to span the steepness range across which separation occurs for both the smooth and rough surfaces. For the smooth surfaces the maximum hill slopes ranged from 0.2 to 0.6 and for the rough surface, where separation occurs at lower slopes, 0.2 to 0.4. Measurements were made in several terrain following transects above each hill for each of the seven wind tunnel runs.

Linear (MSFD) and nonlinear (RaptorNL) model runs were made for each of the slopes and each of the surface roughnesses, for comparison with the wind tunnel measurements. Measurements and model output were scaled using $L$ (the half length of the hill) and $z_0$ (the surface roughness). Here we note that using $z_0/L$-scaling, for a hill of roughly 1 km in length, the rough and smooth surfaces modelled in the wind tunnel are equivalent to tall grass/small scattered shrubs and a 10m tall medium density forest canopy respectively. Figures 3-5 show streamwise transects of the speedup $\Delta S = (U-U_0)/U_0$ where $U_0$ is the upstream unperturbed value of the streamwise component of the flow) measured and modelled at the top of the inner layer ($z=l$). This is near the height of maximum speedup. The open diamonds are wind tunnel measurements, the filled squares are linear model results and the filled triangles are results from the nonlinear RaptorNL model. The solid line along the base of the plot is the vertically
exaggerated hill profile. Plots from the 0.3 slope smooth and rough (0.3S and 0.3R) wind tunnel cases are shown along with the 0.4 slope rough surface run (0.4R).

Figure 3  Streamwise transects of measured and modelled speedup $\Delta S$, at the top of the inner layer for the 0.3S (sand surface, maximum slope = 0.3) case from Ayotte and Hughes (2004). Open diamonds are wind tunnel measurements, filled squares are linear model results and filled triangles are results from the nonlinear Raptor_NL model. The solid line along the base of the plot is the vertically exaggerated hill profile.

Figure 4  As Figure 3 for 0.3R (peg surface, maximum slope = 0.3) case.

Figure 5  As Figure 3 for 0.4R (peg surface, maximum slope = 0.4) case.
It is clear from Figures 3-5 that the nonlinear model performs better than its linear counterpart in all regions of the flow, particularly at the hill crest which is of most interest in siting wind turbines. However, Figure 5 also shows that even the nonlinear solution recovers too quickly in the lee of the hill. We will return to this below. Figures 6 & 7 show the maximum speedup ($\Delta S$) at the top of the inner layer as a function of maximum hill slope for the smooth and rough surfaces respectively. Linear theory and numerical calculations are shown as a solid straight line and filled square symbols respectively. Wind tunnel measurements are shown as open diamonds with a quadratic best-fit dashed line fitted to the measurements. Nonlinear (Raptor$_{NL}$) results are shown as filled triangles. In the context of wind energy calculations, these comparisons suggest that linear calculations can produce significant errors in mean wind estimates if used over terrain with slopes greater than roughly 0.2.

Ayotte and Hughes (2004) proposed two reasons for the discrepancy between linear and nonlinear calculations, both acting in a similar sense to increase the difference between linear theory/modelling and measurements in steep terrain. The first was the obvious difference between the linear and nonlinear advection formulations. This is addressed in the nonlinear model formulation by including the nonlinear advection terms (as well as the weaker nonlinear...
terms in the turbulence closure). The second was the middle and outer layer response to the aggregate shape of the hill in the presence strong separation in the mean flow on the lee side of the hill. To the extent that strong regions of separation can be represented in the mean solution, this has also been addressed by using a nonlinear formulation.

Though nonlinear model formulations show significantly improved results in all regions of the flow (windward, hill crest and leeward) for moderate slopes of up to approximately 0.4 for smooth surfaces and 0.3 for rough surfaces, it appears that there is still significant deviation from measurement on the leeward side for hills with slopes greater than this. This is particularly true for rougher surfaces and can be seen more clearly in Figure 5 where the nonlinear modelled mean flow recovers to near upstream speeds well before the measurements. Ayotte and Hughes (2004) noted that in the lee of even modest slopes, there was a significant increase in turbulent stresses, greater than could be explained by simple mean strain production as represented in standard Reynolds averaged turbulence closures. They went on to suggest that in reality eddies formed in the lee of the hill have time and length scales much larger than those represented in a Reynolds averaged model. These eddies may strongly affect the mean flow well downstream of the hill. As such they suggested an opportunity for the use of eddy resolving calculations such as Large Eddy Simulation (LES) to explore this mechanism. The beginnings of this line of investigation are presented below.

4 RECENT DEVELOPMENTS IN FLOW MODELLING FOR WIND ENERGY CALCULATIONS

4.1 Eddy resolving flow modelling

As we have seen, the performance of a Reynolds averaged model in the lee of a topographic feature of moderate roughness and/or steepness, can be significantly degraded. Locating wind turbines in the lee of a topographic feature where the speed of the mean flow is significantly decreased is unlikely. As such, at first glance it would appear that the above noted deviation between modelled and measured mean flow in the lee of a hill can safely be ignored. Wind tunnel measurements and anecdotal evidence from the wind industry suggest that this is in fact not the most prudent approach. Though mean flow calculations are the primary focus in many cases, when doing wind energy work the effects of strong turbulence are also important when considering turbine design specifications as well as turbine operation and maintenance costs. Understanding these effects and their origins is therefore a worthy objective.

Wood (2000), in a summary of theory and modelling of turbulent flow over hills, included an estimate of the parameter space that could be modelled using a Large Eddy Simulation (LES) model while sufficiently resolving the finer scale turbulent motions within the inner layer of the flow. One of the conclusions based on these estimates was that for a fixed hill length the problem became less computationally expensive over a rougher surface, as the inner layer became deeper and therefore at least somewhat easier to resolve. This effect was captured in a decreasing dimensionless parameter $\lambda/z_0$, where $z_0$ was the surface roughness length and $\lambda$ was the wavelength of the hill. Brown et al. (2000) showed Wood’s estimates to be somewhat pessimistic and carried out calculations simulating the experimental data of Athanassiadou and Castro (1999) from wind tunnel flow over rough sinusoidal 3-D hills ($\lambda/z_0 = 980$). They used the model of Mason and Wood (1993) utilizing a Smagorinsky subgrid model with stochastic
backscatter (Mason and Thomson, 1992). These simulations showed that aerodynamically rough turbulent flow over hills was within the reach of eddy resolving calculations, at least in some segment of the $\lambda/\delta_0$ parameter space. In a step toward tackling more realistic situations with LES, Chow and Street (2004) presented a large eddy simulation of flow over Askervein hill using a modified subgrid scale closure intended to improve the performance of the model in simulating turbulent flow over real terrain. More recently Chow et al. (2006) presented results from eddy resolving calculations at somewhat lower resolution than any of the above, over a mountain valley in the Alps of southern Switzerland. From this work it appears that despite progress toward using LES for real-world atmospheric conditions over nonidealised terrain, direct use of LES for wind energy applications is still some time off. However, stepping back, at least temporarily, from the notion of using LES directly over real terrain, it seems likely that progress can be made by employing LES models to understand the fundamentals of turbulent flow over topography. In particular this would focus on the creation and downwind transport of eddies shed in the lee of topographic features.

Figure 8 Instantaneous streamwise component of the flow from LES over a smooth hill corresponding to Ayotte and Hughes (2004) 0.2S case with $\lambda/\delta_0 = 10^4$. No separation in the lee of the hill is present at this instant.

Figure 9 As Figure 8 for a different time showing strong instantaneous separation on the lee of the hill.
Sullivan et al. (2000) developed and used a curvilinear coordinate system LES model to simulate turbulent flow over two dimensional idealised water waves. This model is currently being used in work aimed at a more clear understanding of the length and time scales associated with turbulence created and transported downstream during intermittent separation. Figures 8 & 9 show instantaneous vertical cross-sections of the streamwise component of the flow for a simulation of the 0.2S case of Ayotte and Hughes (2004), with $\lambda/z_0 = 10^4$ and a maximum slope of 0.2. The calculation uses 800x200x144 grid points in a 4.0x1.0x1.8 km$^3$ domain. Figure 8 shows an instant when there is little if any separation on the lee side of the hill. In contrast Figure 9 shows an instance of strong separation. These eddies are created and shed from the lee side of the hill, dissipating as they move downstream. Time averaged cross-sections of the LES modelled mean flow (not shown) corresponding to Figures 8 & 9, exhibit no signs of mean separation as expected for relatively smooth surfaces over modest slopes. However, this eddy shedding is consistent with the observations of Ayotte and Hughes (2004) that showed indications of plume-like regions where turbulent moments were strongly enhanced in the lee of a smooth isolated hill with a slope of 0.2. Similar calculations of this flow situation done using a Reynolds averaged model with a full second order closure (Raptor$_{NL}$) yield good agreement with mean flow wind tunnel results (see for example Figures 6 & 7) and agree well with time averaged LES results. However, in strong contrast to wind tunnel measurements and the LES, there is no evidence of increased turbulence in the lee of the isolated hill. In fact, quite the opposite is true with production of turbulent moments diminished in the lee of the hill due to lower values of mean shear near the surface, the only mechanism responsible for turbulent production in Reynolds averaged turbulence closures.

4.2 A view of turbulent eddy creation in the lee of a topographic feature

A simple view of intermittent flow separation in the lee of an isolated hill has parcels accelerating toward a pressure minimum at the hill crest as they move up the windward side of the hill. The parcels then enter an area of adverse pressure gradient on the lee side of the hill. As the parcels traverse the area of adverse pressure gradient, they decelerate and may reverse direction if they do not have enough momentum to move completely through the region of
adverse pressure gradient. Parcels in flow over a rougher surface will have less momentum near the surface and as such will separate more easily than parcels with more momentum moving over a smoother surface. In the same context, steeper slopes will create stronger adverse pressure gradients causing parcels to separate more readily. It is also clear from viewing the animations from which Figures 8 & 9 were taken that there is a strong coupling between eddies in the outer region of the flow and those generated near the surface on the lee side of the hill. Local eddy-scale pressure gradients that originate in the outer layer are imposed at the surface due to the elliptical nature of the pressure solution and add to or subtract from the mean adverse pressure gradient created in the lee of the hill. Where parcels with low momentum encounter an adverse pressure gradient that is the in-phase aggregate of the mean topographically induced pressure gradient and the eddy-scale pressure gradient imposed from the outer layer, the parcel reverses direction. Through continuity this often forces an ejection of slow moving fluid from near the surface into the more rapidly moving flow some distance above the surface, creating strong inhomogeneity in streamwise momentum at levels well above the surface. This is seen in Figure 10 which shows the time averaged Turbulent Kinetic Energy (TKE) normalized by the mean wind speed at the top of the middle layer \(z = h_m\).

In the above discussion, a situation has been highlighted where turbulent eddies are being created and shed in the lee of an isolated topographic feature of modest slope, although no mean flow separation occurs. We have seen that even a linearised Reynolds averaged model will give reasonable results in the context of calculating mean acceleration and deceleration in this situation. However, a Reynolds averaged model has no way of capturing eddy generation and shedding that is clearly evident in wind tunnel data and LES results. The reason for this is that the turbulence closure in the Reynolds averaged model can only respond to mean strain, at length scales contained therein. This is in stark contrast to processes that are instantaneously forced at outer layer turbulent eddy length and time scales in combination with the hill length scale. The length scale in a Reynolds averaged turbulence closure is typically determined implicitly by a dissipation equation if it is not simply specified as some height above the surface, for example \(\kappa(z+z_0)\) (see Ayotte et al., 1994). The hill length finds its way into the modelled response only through enhanced shear and dissipation at the top of a surface-based zone of mean velocity deficit in the lee of a hill. This deficit may or may not be strong enough to result in mean flow separation. In either case, at best the modelled turbulent length scale is one associated with a thin shear layer at the top of the sheltered area in the lee of the hill. This is inconsistent with what is suggested by wind tunnel measurements and LES where both outer layer eddy length scales and hill length scales are present or may even dominate. For hills of modest slope, the processes associated with these length scales do not play a large enough role in the vertical transport of momentum to significantly affect the mean flow. However, as is suggested by the wind tunnel results for the 0.4R case (see Figure 5) they can be significant when separation is stronger over steeper slopes. In the case shown in Figure 5 (0.4R) it appears that eddies are created with sufficiently large length and time scales that they propagate some distance downwind before dissipation. These eddies affect the vertical transport of momentum for several hill lengths downstream of the hill – an effect not replicated in either of the Reynolds averaged models.

4.3 Canopy flow modelling

In most modeling efforts to date, the lower momentum boundary condition is treated by specifying a relationship between the speed near the surface, the surface friction velocity and the surface roughness length: \(U(z) = u* \sqrt{k \ln((z+z_0)/z_0)}\) where \(U(z)\) is the wind speed at a height \(z\) above the surface, \(u*\) is the friction velocity and \(z_0\) is a roughness length. In some cases where the
roughness sublayer is a significant depth such as the case of a forest canopy, a displacement height \((d)\) is included in the formulation: \(U(z)\approx u^*/\kappa \ln((z-d+z_0)/z_0)\). This can be seen in Figure 11 which shows idealised wind profiles over a canopy using roughness boundary conditions without and with a displacement height. In this formulation, it is assumed that momentum is absorbed through an infinitely thin layer at a height of \(z_0\) or \(z_0+d\). In both of these formulations, the mean wind profile is logarithmic within and above the vegetative canopy. However, it is evident from measurements that vertical mean wind profiles over a vegetative canopy depart significantly from being logarithmic even over flat ground (see Raupach et al., 1996 and others cited therein). Figure 12 shows a more realistic mean wind profile over a canopy drawn after Raupach et al. (1996). This is shown in direct contrast to a roughness boundary condition with a displacement height. The profile shows a strong departure from a logarithmic profile up to several canopy heights. For a canopy of even modest depth, this is a region of the flow of particular interest in wind farm design as it can span the vertical extent of the turbine disc for even a large turbine.

As noted above, much of the effort in model development in recent times has been aimed at expanding the parameter space over which models used in wind resource assessment will give accurate solutions. One such area where progress is being made is in the development of distributed drag parameterisations in modelling vegetative canopy flows (see for example Ayotte et al., 1999, Finnigan and Belcher, 2004, Ross and Vosper, 2005). In these formulations, momentum is absorbed through a layer of finite thickness rather than through a surface as is for the case of a roughness length \((z_0\) or \(z_0-d\)) parameterisation. This is accomplished by the addition of a drag term in the mean momentum equations,

\[
\frac{\partial U_i}{\partial t} = \ldots - c_d A(z) U_i |U|
\]

where \(c_d\) is a drag coefficient, \(A(z)\) is the foliage area per unit volume. This treatment of drag can also be applied to the second moment equations as in Ayotte et al. (1999).

Figure 11  Standard roughness \((z_0)\) and roughness with displacement height \((z_0+d)\) treatments of mean wind profiles within and above vegetative canopies.
Finnigan and Belcher (2004) showed, using an analytical model, that the effects of a canopy on flow over topography included a diminished hill crest speedup relative to one calculated using a simple roughness parameterisation. Similar results were shown by Ross and Vosper (2005) in fully nonlinear calculations. As with much of the early work on turbulent flow over hills, this work was aimed at least in part at understanding the effects of forest canopies on large scale orographic drag. In addition modelling of this type is important in understanding transport of scalars within and above forest canopies for upscaling canopy flux measurements.

Based on the above work and unpublished work using the RaptorC (the RaptorNL model including the canopy closure from Ayotte et al., 1999), it appears that at least mean flow calculations over hilly terrain with canopies can be undertaken over a reasonably broad range of steepness, canopy depth and density. However, as with flow over unforested hills like that shown in the previous section, the length scales in the flow over topography are poorly represented when using a closure model. The problem is exacerbated somewhat by the presence of canopy length scales. Some authors have tried to address this by the explicit imposition of a canopy length within turbulence closures in Reynolds averaged models. Alternatively a canopy length scale can be implicitly included via a dissipation equation incorporating canopy drag such as done in Ayotte et al. (1999). However, it is unclear that this has provided a solution to the problem of representing canopy length scales in Reynolds averaged model solutions. Finnigan and Belcher (2004) also noted that slow moving canopy flow is even more prone to reversal on the lee side of a topographic feature. As in the previous section we have turned to LES to increase our understanding of the processes associated with turbulent flow in forested terrain.

Following on from work on LES in and above forest canopies over flat ground (Patton, 1997, Patton et al., 2001), Patton et al. (2006) modified the code of Sullivan et al. (2000) to include drag forces associated with a vegetative canopy,

$$\frac{\partial u_i}{\partial t} = -c_d a_U u_i$$

(4)
where $c_d$ is an isotropic drag coefficient, $a$ is a leaf area density and $U$ is the wind speed $(u_H)^{1/2}$. As was the case for flow over a hill without a canopy, the flow is intermittently separated in the lee of the hill, even for small slopes. Figure 13 shows an instantaneous cross section of the streamwise velocity component from a calculation using 800x200x144 grid points in a 4.0x1.0x1.8 km$^3$ domain. Flow is from left to right with reversed flow shaded dark near the surface. The top of the canopy is shown by a dashed line. As with the simulation without canopy drag presented above, there is strong intermittent reversal of the flow with length scales imposed by a combination of outer layer eddy-scale and hill-scale pressure perturbations. Additionally, the figure shows the influence of drag within the canopy where speeds are significantly reduced by canopy drag. Also evident is strong spatial variation in the streamwise component of the flow in the canopy-scale turbulence created by the interaction between the canopy and the free moving flow just above the canopy.

Figure 14 shows the time averaged cross-section of turbulent kinetic energy (TKE) for the same flow. A plume-like region of high levels of TKE is clearly evident in the lee of the hill extending well downwind (in fact wrapping around on the upstream side of the domain – a known limitation of this simulation). Over flat ground, the unstable nature of the strongly sheared flow at the top of the canopy appears to be generating large eddies containing length scales associated with the canopy depth. This is consistent with the mechanism described by Raupach et al. (1996) in which rapidly growing modes of K-H waves generated in the shear at the top of the canopy break, forcing faster moving flow into the canopy and slower moving air upwards to mix with faster moving air above. Additionally, as with the no-canopy case, large eddies containing significant energy are generated in the lee of the hill. These eddies again scale with the hill length and outer layer turbulence pressure gradients. However, production of canopy-scale eddies is significantly enhanced in the lee of the hill with strong interactions between mechanisms that would act to create turbulent motions in the absence of a canopy.

Given that these eddies extend to several canopy heights and contain significant energy, they represent a potential source of wind turbine damage or premature wear in the lee of canopied slopes or as other simulations have shown, possibly over flat ground. The Reynolds averaged model results of Ross and Vosper (2005) and unpublished work using the RaptorC model, did not show this extensive downwind propagation of hill/canopy-scale turbulent eddies. This appears to be a fundamental and significant shortcoming in Reynolds averaged models and an area where development effort may be well spent.
Figure 13 Instantaneous streamwise component of the flow from LES over a rough (peg surface) hill corresponding to Ayotte and Hughes (2004) 0.2R case with \( \lambda/\zeta_0 = 10^3 \). Canopy depth is shown as dashed line. Slow moving and reversed flow is clearly evident within the canopy and up to several canopy heights in the lee of the hill.

Figure 14 Time averaged turbulent kinetic energy normalised by the velocity at the top of the middle layer for LES show in Figure 13. Dark shades show low values within the canopy and high values in a plume-like region extending downwind in the lee of the hill.

5 CONCLUSIONS

Though by no means complete or particularly rigorous, what has been presented here is an overview of progress made in numerical computations used for wind farm design, primarily in wind resource assessment. We have seen an evolution from linear models through to nonlinear Reynolds averaged models and we are now beginning to see eddy resolving models such as LES in limited use within the wind industry. Nonlinear Reynolds averaged models show high levels of skill in predicting the mean flow upon which energy yield calculations can be made, but these models are clearly lacking in their ability to explicitly yield predictions of turbulence. This is due to fundamental limitations associated with the Reynolds averaged formulation. If we are seeking to calculate only the mean flow, this inability can in some instances be unimportant. However, where turbulent motions strongly affect the mean or where we are explicitly interested in turbulence predictions, there is a gap between our requirements and our modelling abilities, at least in the context of what is currently commercially available.

It was noted earlier that for the foreseeable future, our modelling efforts are unlikely to result in a model that can simultaneously resolve the full range of atmospheric length and time scales. It was also noted that despite this, resolved length and time scales in meteorological modelling are decreasing. Continuation of this trend will require more computational power and perhaps a new view of how these computations are used within the industry. Given recent advances in multi-core chip technology and other as yet to be imagined advances, it seems likely that some sort of eddy resolving computations will eventually make their way into the mainstream of wind farm design. This may be within the European Wind Atlas methodology or some other hybrid methodology in which mesoscale information is assimilated into finer scale calculations. Also, as competence and confidence in high resolution calculations grows, using output from eddy resolving calculations as input for aeroelastic turbine design calculations seems a natural direction to take. To a limited extent this has already begun with work such as Kelly et al.,
(2004) using LES output from simulations of breaking K-H waves at the top of a shallow boundary layer as input for turbine design models. Moving further in this direction, if accurate turbulence spectra can be reproduced within simulations of turbulent flow over real topographic features, this represents a rich source of information for use in wind turbine design.

REFERENCES